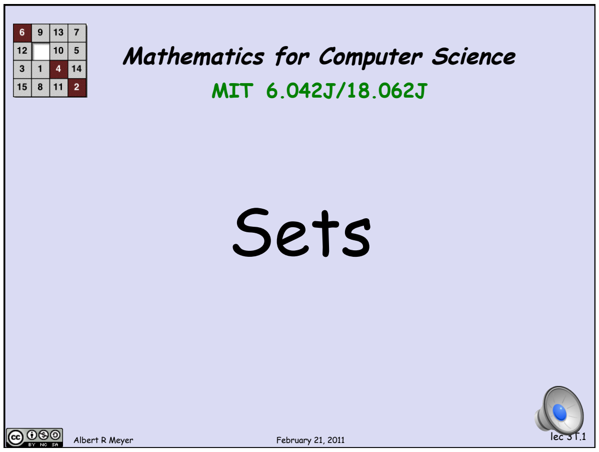
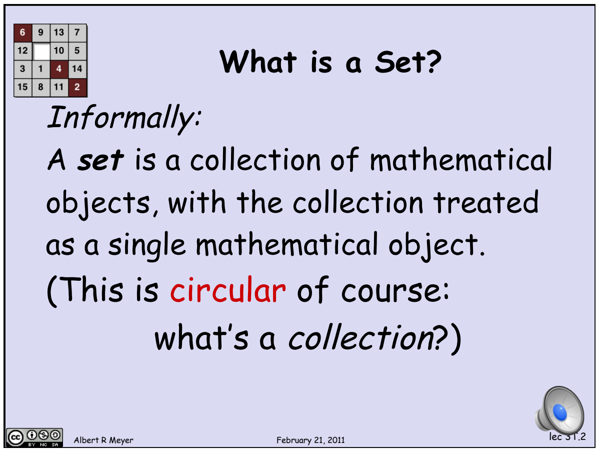


Mathematics for Computer Science

MIT 6.042J/18.062J

Sets

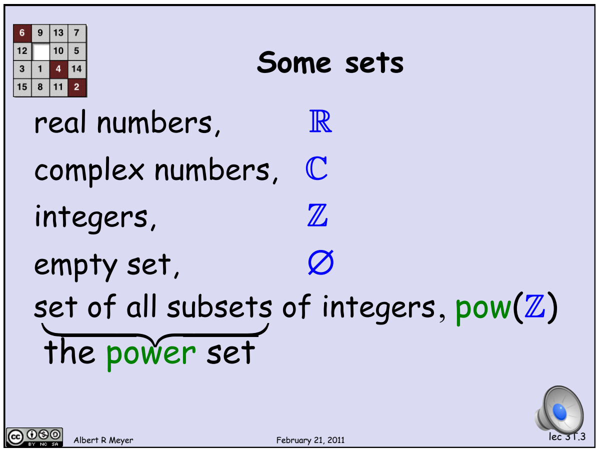


What is a Set?

Informally:

A ***set*** is a collection of mathematical objects, with the collection treated as a single mathematical object.

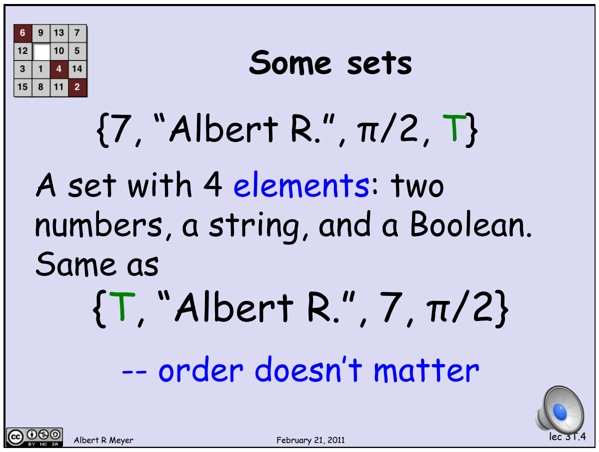
(This is circular of course: what's a *collection*?)



Some sets

real numbers,	$\mathbb{R}$
complex numbers,	$\mathbb{C}$
integers,	$\mathbb{Z}$
empty set,	$\varnothing$

$\underbrace{\text{set of all subsets}}_{\text{the power set}}$ of integers, pow($\mathbb{Z}$)



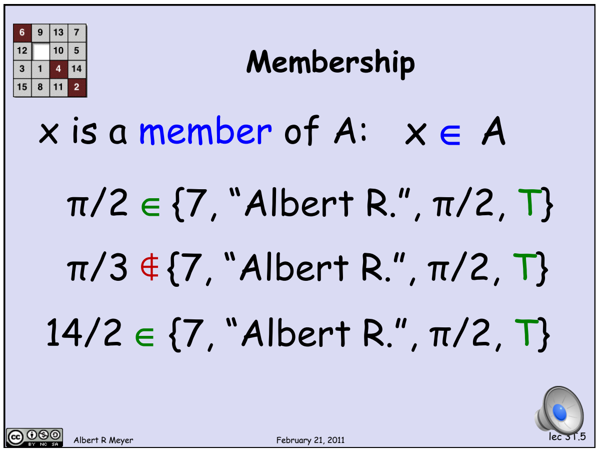
Some sets

$\{7, \text{“Albert R.”}, \pi/2, T\}$

A set with 4 elements: two numbers, a string, and a Boolean. Same as

$\{T, \text{“Albert R.”}, 7, \pi/2\}$

-- order doesn't matter



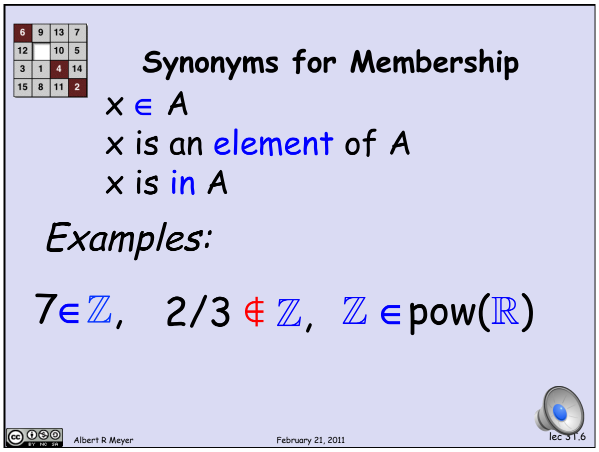
Membership

x is a member of A: $x \in A$

$\pi/2 \in \{7, \text{“Albert R.”}, \pi/2, T\}$

$\pi/3 \notin \{7, \text{“Albert R.”}, \pi/2, T\}$

$14/2 \in \{7, \text{“Albert R.”}, \pi/2, T\}$



Synonyms for Membership

$x \in A$

x is an element of A

x is in A

Examples:

$7 \in \mathbb{Z}$, $2/3 \notin \mathbb{Z}$, $\mathbb{Z} \in \text{pow}(\mathbb{R})$

In or Not In

An element is in or not in a set:

$\{7, \pi/2, 7\}$ is same as $\{7, \pi/2\}$

(No notion of being in the set more than once)

Subset ($\subseteq$)

$A \subseteq B$ A is a subset of B
A is contained in B

Every element of A is also an element of B:

$\forall x\ [x \in A\ \text{IMPLIES}\ x \in B]$

Subset

examples:

$\mathbb{Z} \subseteq \mathbb{R}$, $\mathbb{R} \subseteq \mathbb{C}$, $\{3\} \subseteq \{5,7,3\}$

$A \subseteq A$, $\varnothing \subseteq$ every set

$\varnothing \subseteq$ everything

def: $\varnothing \subseteq B$

$$\forall x\ \underbrace{[\underbrace{x \in \varnothing}_{\text{false}}\ \text{IMPLIES}\ x \in B]}_{\text{true}}$$

Defining Sets

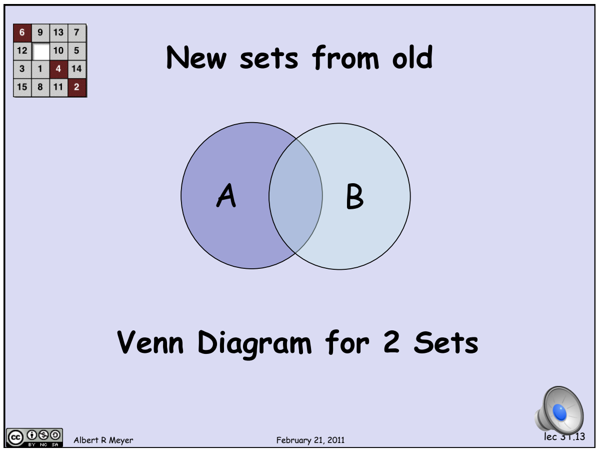
The set of elements, x, in A such that P(x) is true.

$$\{x \in A \mid P(x)\}$$

Defining Sets

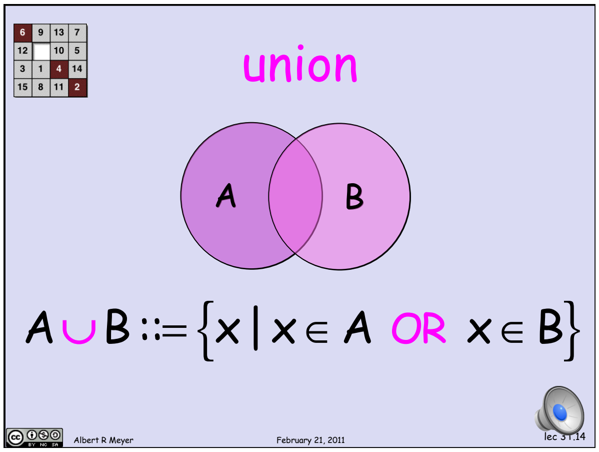
The set of even integers:

$$\{n \in \mathbb{Z} \mid n \text{ is even}\}$$



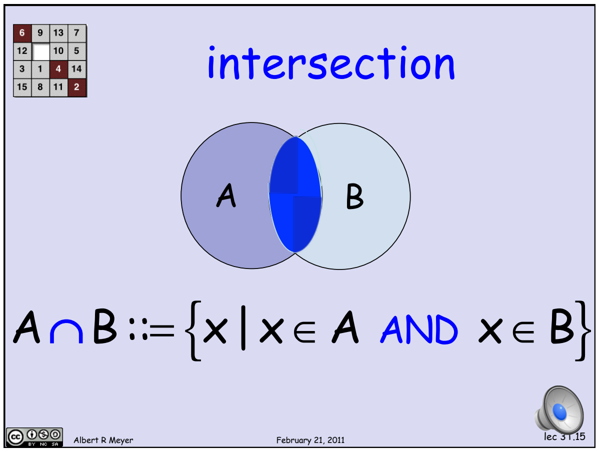
New sets from old

Venn Diagram for 2 Sets



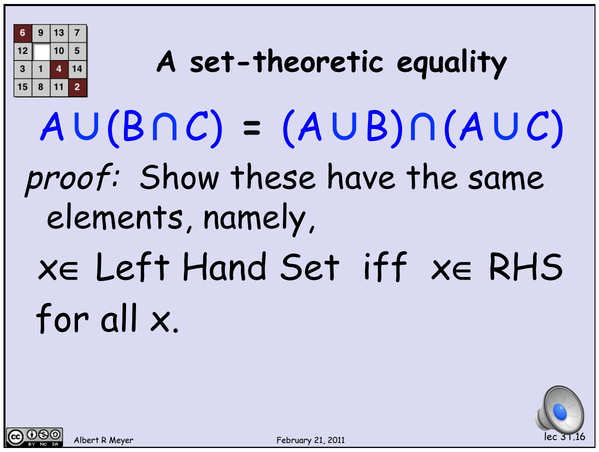
union

$$A \cup B ::= \{x \mid x \in A \text{ OR } x \in B\}$$



intersection

$$A \cap B ::= \{x \mid x \in A \text{ AND } x \in B\}$$



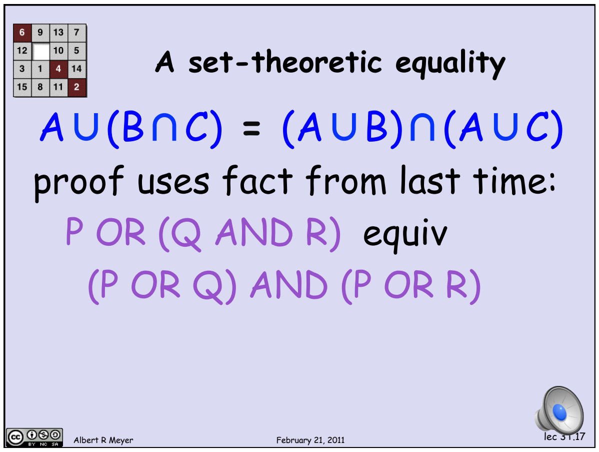
A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof: Show these have the same elements, namely,

$x \in$ Left Hand Set iff $x \in$ RHS

for all x.



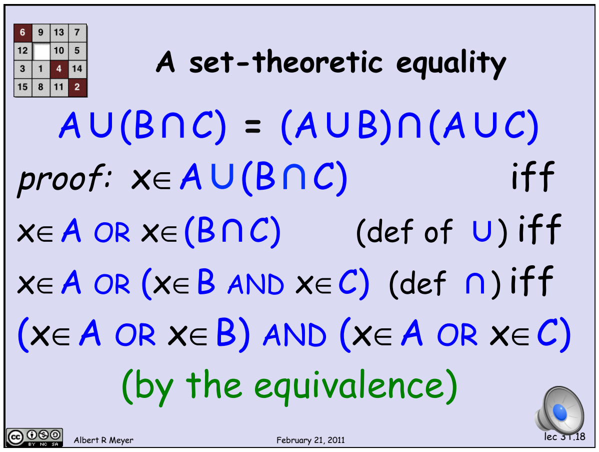
A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

proof uses fact from last time:

P OR (Q AND R) equiv

(P OR Q) AND (P OR R)



A set-theoretic equality

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

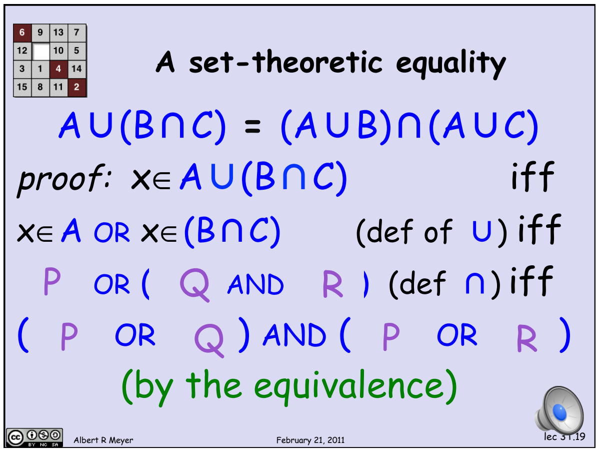
proof: $x \in A \cup (B \cap C)$ iff

$x \in A$ OR $x \in (B \cap C)$ (def of $\cup$) iff

$x \in A$ OR ($x \in B$ AND $x \in C$) (def $\cap$) iff

($x \in A$ OR $x \in B$) AND ($x \in A$ OR $x \in C$)

(by the equivalence)



A set-theoretic equality

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

proof: $x \in A \cup (B \cap C)$ iff

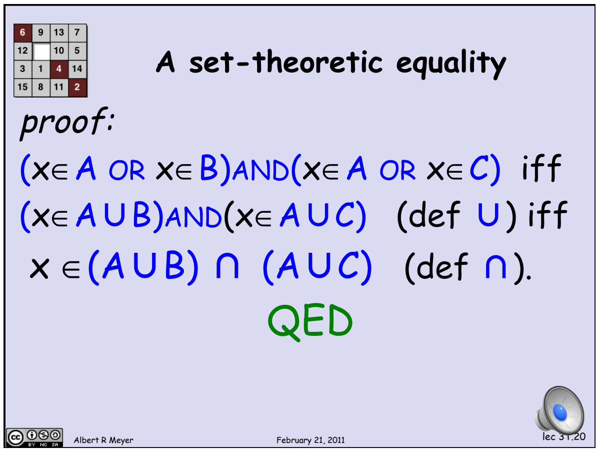
$x \in A$ OR $x \in (B \cap C)$ (def of $\cup$) iff

P OR (Q AND R) (def $\cap$) iff

(P OR Q) AND (P OR R)

(by the equivalence)

Albert R Meyer February 21, 2011 lec 3T.19



A set-theoretic equality

proof:

$(x \in A$ OR $x \in B)$ AND $(x \in A$ OR $x \in C)$ iff

$(x \in A \cup B)$ AND $(x \in A \cup C)$ (def $\cup$) iff

$x \in (A \cup B) \cap (A \cup C)$ (def $\cap$).

QED

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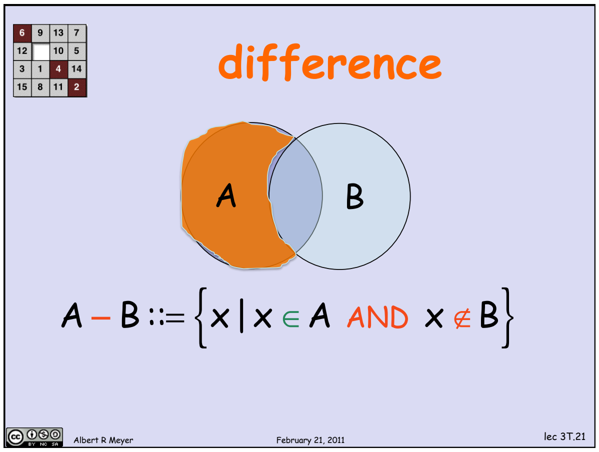

difference

$A - B ::= \{x \mid x \in A$ AND $x \notin B\}$

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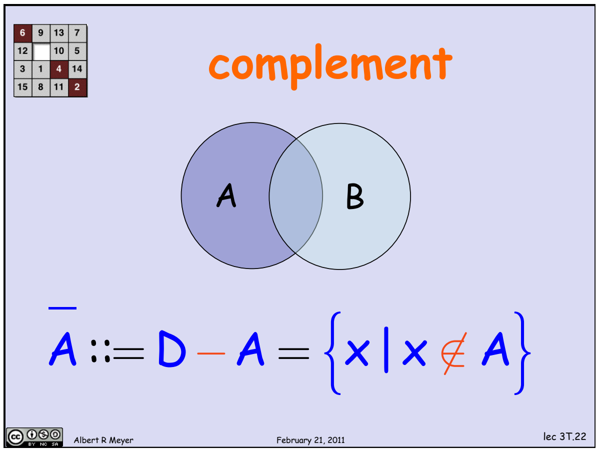

complement

$\overline{A} ::= D - A = \{x \mid x \notin A\}$

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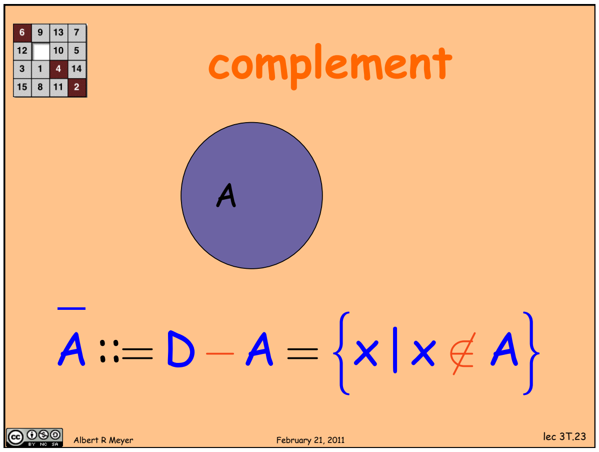

complement

$\overline{A} ::= D - A = \{x \mid x \notin A\}$

Albert R Meyer February 21, 2011 lec 3T.23