

Recursive function on M
Def. tree-depth(s) for
$$s \in M$$

 $td(\lambda)$::= 0
 $td([s]t)$::=
 $1 + max{td(s), td(t)}$



Recursive Function

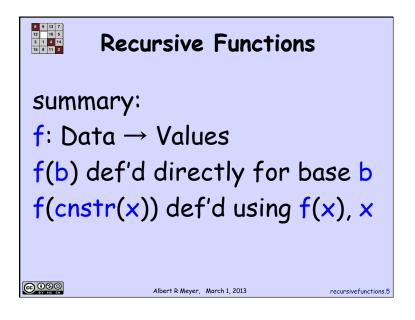
To define a function, f, on a recursively defined set R, define •f(b) explicitly for each base case $b \in R$

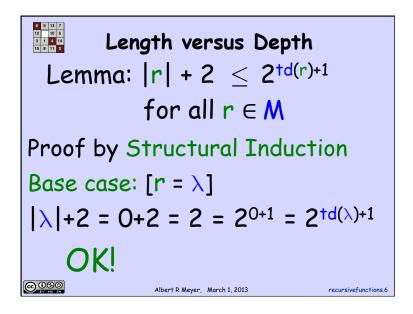
•f(c(x)) for each constructor, c,
in terms of x and f(x)

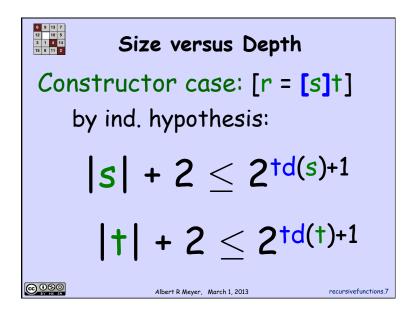
Albert R Meyer, March 1, 2013

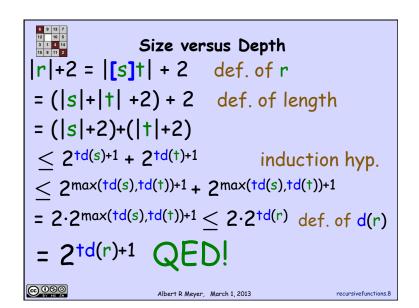
recursivef

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\begin{array}{l} \overbrace{k^{n}} & -\operatorname{recursive function on } N \\ expt(k, 0) & ::= 1 \\ expt(k, n+1) ::= k \cdot expt(k,n) \\ -\operatorname{uses recursive def of } N: \\ -\operatorname{uses recursive def of } N: \\ \circ & 0 \in \mathbb{N} \\ \circ & \operatorname{if} n \in \mathbb{N}, \ \operatorname{then} n + 1 \in \mathbb{N} \end{array}
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positive powers of two

$$2 \in PP2$$

if x,y $\in PP2$, then x \cdot y $\in PP2$
2, 2 \cdot 2, 4 \cdot 2, 4 \cdot 4, 4 \cdot 8, ...
2 4 8 16 32 ... $\in PP2$

$$loggy function on PP2$$

$$loggy(2)::= 1$$

$$loggy(x \cdot y) ::= x + loggy(y)$$

$$for x, y \in PP2$$

$$loggy(4) = loggy(2 \cdot 2) = 2 + 1 = 3$$

$$loggy(8) = loggy(2 \cdot 4) = 2 + loggy(4)$$

$$= 2 + 3 = 5$$

$$loggy(16) = loggy(8 \cdot 2) = 8 + loggy(2)$$

$$= 8 + 1 = 9$$

$$log_{2} \text{ of } PP2$$

$$log_{2}(2) ::= 1$$

$$log_{2}(x \cdot y) ::= log_{2}(x) + log_{2}(y)$$
for x, y \in PP2

$$log_{2}(4) = log_{2}(2 \cdot 2) = 1 + 1 = 2$$

$$log_{2}(8) = log_{2}(2 \cdot 4) = log_{2}(2) + log_{2}(4)$$

$$= 1 + 2 = 3$$

$$loggy function on PP2$$

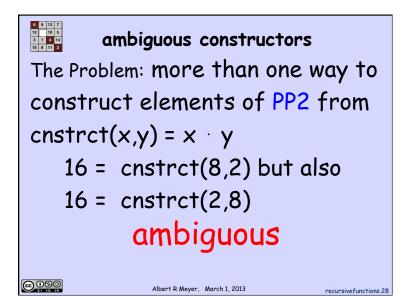
$$loggy(16) = loggy(8 \cdot 2) = 9$$

$$WAIT \ A \ SEC!:$$

$$loggy(16) = loggy(2 \cdot 8)$$

$$= 2 + loggy(8) = 2 + 5$$

$$= 7$$



ambiguous recursive defs problem to watch out for: recursive function on datum, e, is defined according to what constructor created e. If 2 or more ways to construct e, then which definition to use?

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