

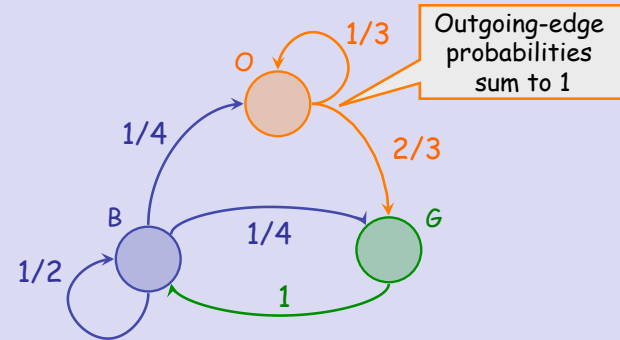
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

# Random Walks



6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

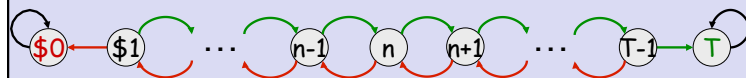
## Graph With Probable Transitions



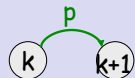
6	9	13	7
12		10	5
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15	8	11	2

## Example: Gambler's Ruin

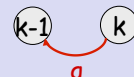
View as random walk on a line.



$p ::= \Pr[\text{win a bet}]$



$q ::= 1-p = \Pr[\text{lose a bet}]$



What is  $\Pr[\text{reach } T \text{ before } 0]$ ?



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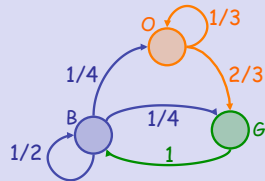
## Applications of Random Walk

- Physics — Brownian motion
- Finance — stocks, options
- Algorithms — web search, clustering



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Questions



- $\Pr[\text{reach } O \text{ in } 7 \text{ steps} \mid \text{start at } B]$
- Average # steps from B to O
- $\Pr[\text{reach } G \text{ before } O \mid \text{start at } B]$



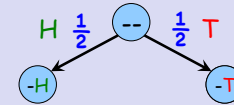
Albert R Meyer,

May 13, 2015

random-walk.5

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Example: Toss HTH before TTH



$$\begin{aligned} \Pr[\text{win}] &= \Pr[\text{win} \mid \text{--}] \\ &= \frac{1}{2} \Pr[\text{win} \mid \text{-H}] + \frac{1}{2} \Pr[\text{win} \mid \text{-T}] \end{aligned}$$



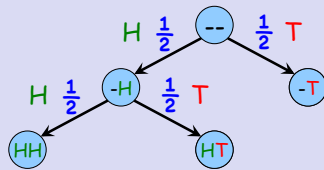
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random-walk.6

6	9	13	7
12	10	5	
3	1	4	14
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## Example: Toss HTH before TTH



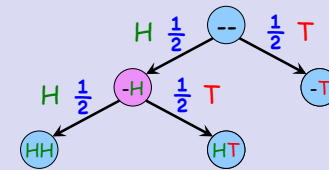
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random-walk.7

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Example: Toss HTH before TTH



$$\begin{aligned} \Pr[\text{win} \mid \text{-H}] &= \frac{1}{2} \Pr[\text{win} \mid \text{HH}] \end{aligned}$$



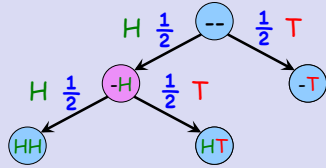
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random-walk.8

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: Toss **HTH** before **TTH**



$$\Pr[\text{win} | \text{-H}] = \frac{1}{2}\Pr[\text{win} | \text{HH}] + \frac{1}{2}\Pr[\text{win} | \text{HT}]$$

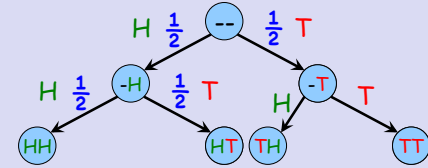


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random-walk.9

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: Toss **HTH** before **TTH**

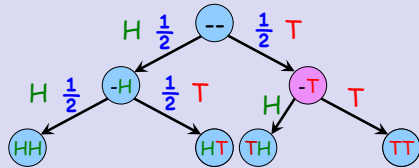


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random-walk.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Example: Toss **HTH** before **TTH**



$$\Pr[\text{win} | \text{-T}] = \frac{1}{2}\Pr[\text{win} | \text{TH}] + \frac{1}{2}\Pr[\text{win} | \text{TT}]$$

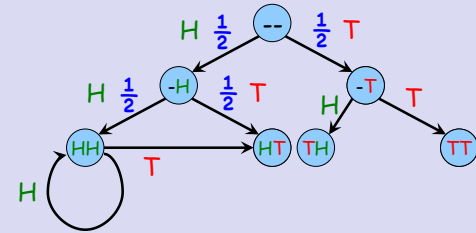


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random-walk.11

6	9	13	7
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Example: Toss **HTH** before **TTH**



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random-walk.12

6	9	13	7
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### Example: Toss **HTH** before **TTH**

$\Pr[\text{win} | \text{HH}]$   
 $= \frac{1}{2}\Pr[\text{win} | \text{HH}] + \frac{1}{2}\Pr[\text{win} | \text{HT}]$

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6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

### Example: Toss **HTH** before **TTH**

$\Pr[\text{win} | \text{win}] = 1$      $\Pr[\text{win} | \text{lose}] = 0$   
 Now solve system of linear equations for  $\Pr[\text{win}]$

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6	9	13	7
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### Questions

- $\Pr[\text{reach } O \text{ in } 7 \text{ steps} | \text{start at } B]$
- Average # steps from **B** to **O**
- $\Pr[\text{reach } G \text{ before } O | \text{start at } B]$

Just solve systems of linear equations

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