

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
6.042J/18.062J

PROOFS



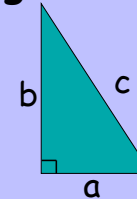
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proof-intro.1

6	9	13	7
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Getting started:
Pythagorean theorem



$$a^2 + b^2 = c^2$$

Familiar? Yes!

Obvious? No!



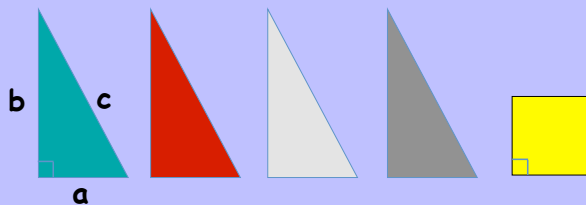
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proof-intro.3

6	9	13	7
12		10	5
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A Cool Proof



Rearrange into:

- (i) a $c \times c$ square, and then
- (ii) an $a \times a$ & a $b \times b$ square



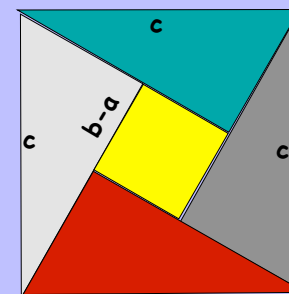
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proof-intro.4

6	9	13	7
12		10	5
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A Cool Proof



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proof-intro.5

6	9	13	7
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A Cool Proof

b c
 a $b-a$

$b-a$

$b-a$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

A Cool Proof

a b

$b-a$ a

$(b-a)+a$

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6	9	13	7
12		10	5
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A Cool Proof

a b

a $b-a$ a

b

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6	9	13	7
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Proof by Picture

- elegant and correct
- in this case
- worrisome in general
- hidden assumptions

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6	9	13	7
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Bogus Proof: Getting Rich By Diagram



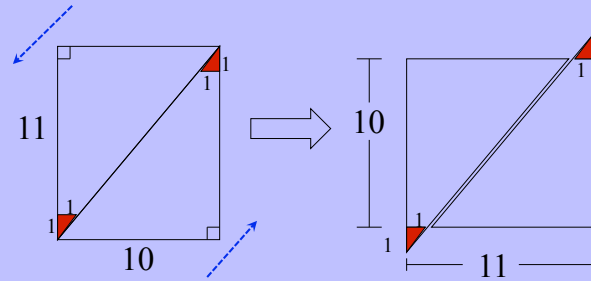
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proof-intro.11

6	9	13	7
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Bogus Proof: Getting Rich By Diagram



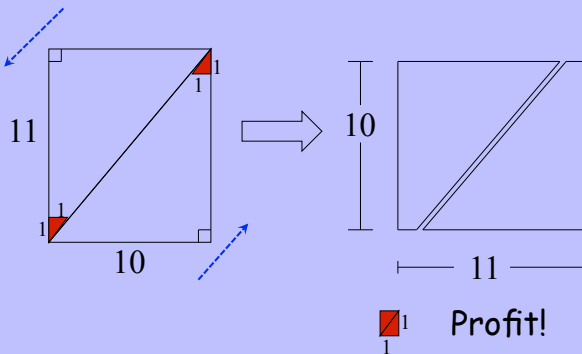
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proof-intro.12

6	9	13	7
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A False Proof: Getting Rich By Diagram



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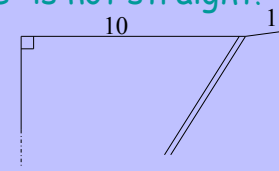
proof-intro.13

6	9	13	7
12		10	5
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Getting Rich

The bug:

\triangle \triangle are not right triangles!
So the top and bottom line of the
"rectangle" is not straight!



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proof-intro.14

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Another Bogus Proof

Theorem: Every polynomial,
 $ax^2 + bx + c$
 has two roots over \mathbb{C} .

Proof (by calculation). The roots are:

$$r_1 ::= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 ::= \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$



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proof-intro.15

6	9	13	7
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Another Bogus Proof

Counter-examples:

$0x^2 + 0x + 1$ has 0 roots

$0x^2 + 1x + 1$ has 1 root

The bug: divide by zero error

The fix: require $a \neq 0$



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proof-intro.16

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Another Bogus Proof

Counter-example:

$1x^2 + 0x + 0$ has 1 root.

The bug: $r_1 = r_2$

The fix: require $D \neq 0$ where

$$D ::= b^2 - 4ac$$



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proof-intro.17

6	9	13	7
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Another Bogus Proof

What if $D < 0$?

$x^2 + 1$ has roots $i, -i$

--ambiguous which is r_1
 and which is r_2 ?



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proof-intro.18

6	9	13	7
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$$1 = -1 ?$$

ambiguity can cause problems:

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = (\sqrt{-1})^2 = -1$$

Moral:

1. Be sure rules are properly applied.
2. Thoughtless calculation no substitute for understanding.



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proof-intro.19

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Consequences of $1 = -1$

$$\frac{1}{2} = -\frac{1}{2} \quad (\text{multiply by } \frac{1}{2})$$

$$2 = 1 \quad (\text{add } \frac{3}{2})$$

*“Since I and the Pope are clearly 2,
we conclude that I and the Pope are 1.
That is, I am the Pope.”*

-- Bertrand Russell



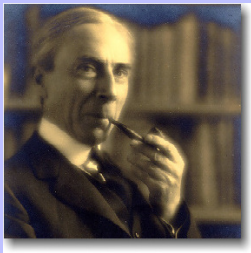
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proof-intro.21

6	9	13	7
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Consequences of $1 = -1$



Bertrand Russell (1872 - 1970)

(Picture source: <http://www.users.drew.edu/~j1anz/brs.html>)



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proof-intro.22