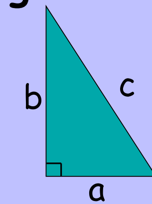
Mathematics for Computer Science
6.042J/18.062J

PROOFS

Getting started: Pythagorean theorem



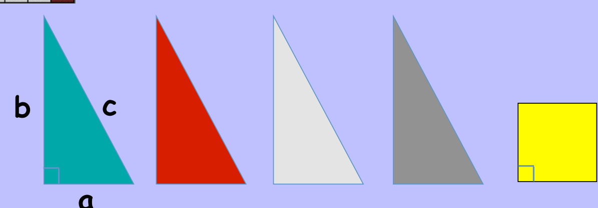
$$a^2 + b^2 = c^2$$

Familiar? Yes!

Obvious? No!

c×c

A Cool Proof

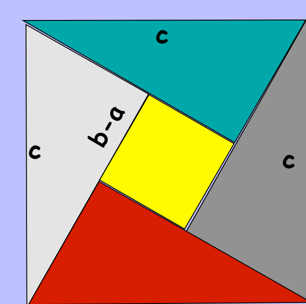


Rearrange into:

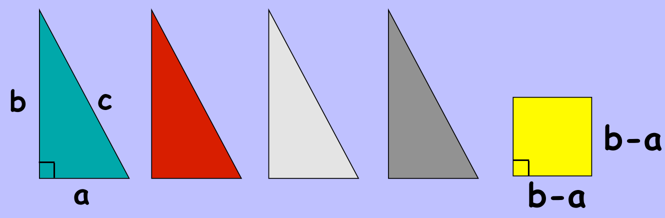
(i) a **c × c** square, and then

(ii) an **a×a** & a **b×b** square

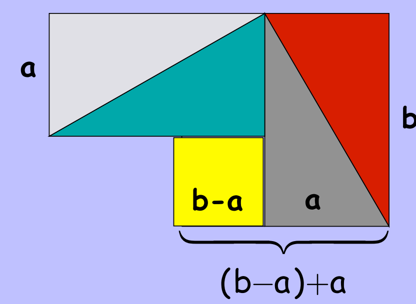
A Cool Proof



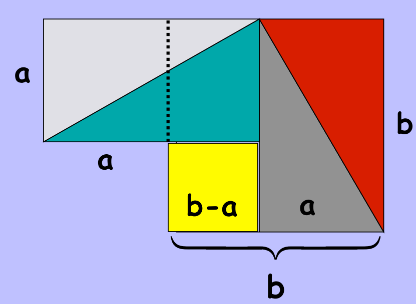
A Cool Proof



A Cool Proof



A Cool Proof

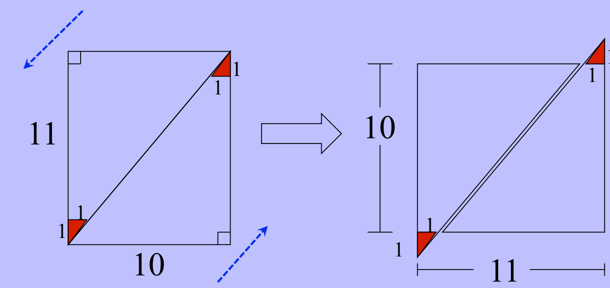


Proof by Picture

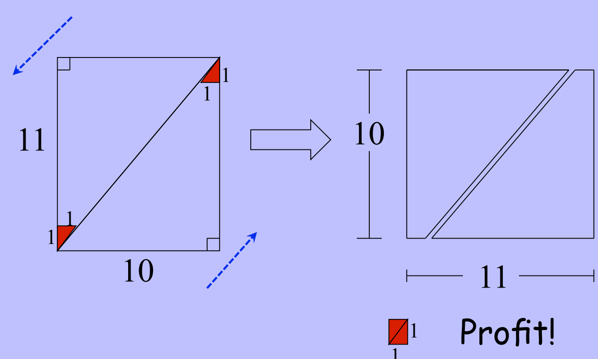
- elegant and correct
  --in this case
- worrisome in general
  --hidden assumptions

Bogus Proof: Getting Rich By Diagram

Bogus Proof: Getting Rich By Diagram

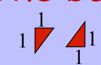


A False Proof: Getting Rich By Diagram



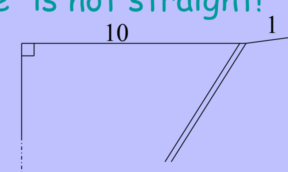
Getting Rich

The bug:



are not right triangles!

So the top and bottom line of the "rectangle" is not straight!



Another Bogus Proof

Theorem: Every polynomial,

$ax^2 + bx + c$

has two roots over $\mathbb{C}$.

Proof (by calculation). The roots are:

$$r_1 ::= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 ::= \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Another Bogus Proof

Counter-examples:

$0x^2 + 0x + 1$ has 0 roots

$0x^2 + 1x + 1$ has 1 root

The bug: divide by zero error

The fix: require $a \neq 0$

Another Bogus Proof

Counter-example:

$1x^2 + 0x + 0$ has 1 root.

The bug: $r_1 = r_2$

The fix: require $D \neq 0$ where

$$D ::= b^2 - 4ac$$

Another Bogus Proof

What if $D < 0$?

$x^2 + 1$ has roots $i, -i$

--ambiguous which is r_1 and which is r_2?

1 = -1 ?

ambiguity can cause problems:

$$1=\sqrt{1}=\sqrt{(-1)(-1)}=\sqrt{-1}\sqrt{-1}=\left(\sqrt{-1}\right)^2=-1$$

Moral:

1. Be sure rules are properly applied.
2. Thoughtless calculation no substitute for understanding.

Consequences of 1 = -1

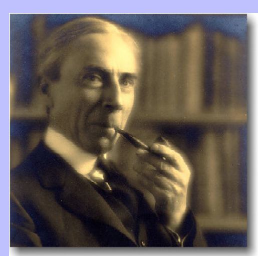
$\frac{1}{2} = -\frac{1}{2}$ (multiply by $\frac{1}{2}$)

$2 = 1$ (add $\frac{3}{2}$)

"Since I and the Pope are clearly 2*, we conclude that I and the Pope are* 1*. That is, I am the Pope."*

-- Bertrand Russell

Consequences of 1 = -1



Bertrand Russell (1872 - 1970)

(Picture source: http://www.users.drew.edu/~jlenz/brs.html)