

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Predicate Logic, III

\forall \exists in English

Two Meta-Theorems



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Poet: "All that G glitters is not Au gold."

~~$\forall x. [G(x) \text{ IMPLIES } \text{NOT}(Au(x))]$~~

No: gold glitters like gold!



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Poet: "All that glitters is not gold."
necessarily
 $\text{NOT}(\forall x. [G(x) \text{ IMPLIES } Au(x)])$
(Poetic license)



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Poet: "There is a season to every purpose under heaven"
 $\exists s \in \text{Season} \forall p \in \text{Purpose}. s \text{ is for } p$
Some season, say Summer, is good for all Purposes?
NO, Summer no good for snow shoveling



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Poet: "There is a season to every purpose under heaven"
 $\exists s \in \text{Season} \forall p \in \text{Purpose}. s \text{ is for } p$
Poet's meaning flips the quantifiers



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Poet: "There is a season to every purpose under heaven"
 $\forall p \in \text{Purpose} \exists s \in \text{Season}. s \text{ is for } p$
Poet's meaning flips the quantifiers



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Math vs. English

Poet: "There is a season to every purpose under heaven"

$\forall p \in \text{Purpose} \exists s \in \text{Season}. s$ is for p
 for snow shoveling, Winter is good
 for planting, Spring is good
 for leaf watching, Fall is good



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Power & Limits of Logic

Two Profound Meta-Theorems about Mathematical Logic



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Gödel's Completeness Theorem

Thm 1, **good news**: only need to know a few axioms & rules to prove **all** valid formulas.
 (in theory; in practice need lots of rules)



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Axioms & Inference Rules

Rules are just UG and modus ponens. Most of the valid axioms shown already.



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Validity is **undecidable**

Thm 2, **Bad News**: there is no procedure to determine whether a quantified formula is valid (in contrast to propositional formulas).



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Profound Meta-Theorems

We won't examine these Theorems further. Their proofs usually require half a term in an intro logic course after 6.042. But they are interesting to think about.



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