Mathematics for Computer Science
MIT 6.042J/18.062J

Predicate Logic, II

Validity & Satisfiability

Propositional Validity

True for all truth-values.
Example:

(P IMPLIES Q) OR (Q IMPLIES P)

Predicate Calculus Validity

True for all domains and predicates. *Example:*

∀z.[P(z) AND Q(z)] IMPLIES
[∀x.P(x) AND ∀y.Q(y)]

Proving Validity

∀z.[P(z) AND Q(z)] IMPLIES
[∀x.P(x) AND ∀y.Q(y)]

Proof strategy: assume left side is T, then prove right side is T

Proving Validity

$\forall z[Q(z) \land P(z)] \rightarrow [\forall x.Q(x) \land \forall y.P(y)]$

Proof: Assume left hand side.That is, for all values of z in the domain, Q(z) AND P(z) is true. Suppose val(z) = c, an element in the domain. Then Q(c) AND P(c) holds, and so Q(c) by itself holds. But c could have been any element of the domain. So we conclude ∀x.Q(x). (by **UG**)
Similarly conclude ∀y.P(y). Therefore,
∀x.Q(x) AND ∀y.P(y) QED

Universal Generalization (UG)

$$\frac{P(c)}{\forall x.P(x)}$$

providing c does not occur in P

Similar Example is Not Valid

$\forall z.[P(z)$ OR $Q(z)]$ IMPLIES

$[\forall x.P(x)$ OR $\forall y.Q(y)]$

Proof: Give *counter-model*, where left side of IMPLIES is T, but right side is F.

Namely, let domain ::= {1, 2},

Q(z) ::= [z = 1], P(z) ::= [z = 2].

DeMorgan's Law for Quantifiers

Another valid formula:

NOT$(\forall x.\ P(x))$ IFF

$\exists y.$ NOT$(P(y))$