Predicate Logic, I
Quantifiers $\forall, \exists$

Predicates
$P(x,y) ::= [x + 2 = y]$

- $x = 1$ and $y = 3$: $P(1,3)$ is true
- $x = 1$ and $y = 4$: $P(1,4)$ is false

$\forall s. P(s)$
same as
$P(Drew) \land P(Peter) \land P(Keshav) \land \ldots \land P(Michaela)$

Quantifiers

$\forall x$ For ALL $x$
$\exists y$ There EXISTS some $y$

$\exists$ is like OR
Let $t$ range over 6.042 staff
$B(t) ::= [t \text{ took 6.042 Before}]$

$\exists t. B(t)$
same as
$B(Drew) \lor B(Peter) \lor B(Keshav) \lor \ldots \lor B(Michaela)$
Existential Quantifier
Let \( x, y \) range over \( \mathbb{N} \)
\[ Q(y) ::= \exists x. x < y \]
\( Q(3) \) is \( \top \) (\([x<3]\) is \( \top \) for \( x=1 \))
\( Q(1) \) is \( \top \) (\([x<1]\) is \( \top \) for \( x=0 \))
\( Q(0) \) is \( \bot \) (\([x<0]\) is not \( \top \) for any \( x \) in \( \mathbb{N} \))

Universal Quantifier
\( x, y \) range over \( \mathbb{N} \)
\[ R(y) ::= \forall x. x < y \]
\( R(1) \) is \( \bot \) (\([x<1]\) is \( \bot \) for \( x=5 \))
\( R(8) \) is \( \bot \) (\([x<8]\) is \( \bot \) for \( x=12 \))
\( R(10^{100}) \) is \( \bot \) (\([x<10^{100}]\) is \( \bot \) for \( x=10^{100} \))

For every virus, I have a defense:
- against MYDOOM, use Defender
- against ILOVEYOU, use Norton
- against BABLAS, use Zonealarm...

\( \forall \exists \) is expensive!

Example: \( d \) is MITviruscan, protects against all viruses

That's what we want!

Alternating Quantifiers
\[ G ::= \forall x \exists y. x < y \]
\( x, y \) range over Domain of Discourse
\( \text{Domain} \) \[ \begin{array}{ll}
\mathbb{N} & \top \\
\text{ints} < 0 & \bot \\
\text{reals} < 0 & \bot 
\end{array} \]
\( G \) is:
\[ \begin{array}{ll}
\mathbb{N} & \top \\
\mathbb{Z}^- & \bot \\
\mathbb{R}^- & \bot 
\end{array} \]

Reverse the Quantifiers
\[ H ::= \exists y \forall x. x \leq y \]
\( \text{Domain} \) \[ \begin{array}{ll}
\mathbb{N} & \bot \\
\mathbb{Z}^- & \top \\
\mathbb{R}^- & \bot 
\end{array} \]
\( H \) is:
\[ \begin{array}{ll}
\mathbb{N} & \bot \\
\mathbb{Z}^- & \bot \\
\mathbb{R}^- & \bot 
\end{array} \]