Midterm Exam February 25

Your name:____

Fill in your Session (**1** or **2:30**): ______PM Table (**A–K**): ______

- This exam is **closed book** except for a 2-sided cribsheet. Total time is 80 minutes.
- Write your solutions in the space provided. If you need more space, **write on the back** of the sheet containing the problem.
- In answering the following questions, you may use without proof any of the results from class or text.

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	25		
2	25		
3	15		
4	10		
5	25		
Total	100		

Problem 1 (An Irrational Number) (25 points).
Prove that $\sqrt[7]{35}$ is irrational.

Problem 2 (Well Ordering Principle) (25 points).
Use the Well Ordering Principle to prove that the equation

$$3a^4 + 9b^4 = c^4$$

has no positive integer solutions.

Problem 3 (Predicate Logic) (15 points).
Express each of the following predicates and propositions in formal logic notation. The domain of discourse is the nonnegative integers, $\mathbb{N}$. Moreover, in addition to the propositional operators, variables and quantifiers, you may define predicates using addition, multiplication, and equality symbols, but no *exponentiation* (like x^y) and no integer *constants* like `0` or `1`.

For example, the predicate "$x \geq y$" could be expressed by the following logical formula.

$$\exists w. (y + w = x).$$

Now that we can express $\geq$, it's OK to use it to express other predicates. For example, the predicate $x < y$ can now be expressed as

$$y \geq x \text{ AND NOT}(x = y).$$

For each of the predicates below, describe a logical formula to express it. It is OK to use in the logical formula any of the predicates previously expressed.

(a) $x = \mathtt{1}$.

(b) m is a divisor of n (notation: $m \mid n$)

(c) n is a prime number.

(d) n is a power of a prime.

Problem 4 (**Binary Relations**) (**10 points**).
Formulas defining functions from integers to integers are listed below. For each function, indicate whether it is

- **B**, a bijection [$= 1$ out, $= 1$ in],
- **S**, surjection [≥ 1 in], but not a bijection,
- **I**, an injection [≤ 1 in], but not a bijection,
- **N**, neither an injection nor a surjection.

(a) $a(x) ::= x^2$. ________

(b) $b(x) ::= x + 2$. ________

(c) $c(x) ::= 2x$. ________

(d) $d(x) ::= -x$. ________

(e) $e(x) ::= \lfloor x/2 \rfloor$, that is, the quotient of x divided by 2. ________

Problem 5 (**Class Teams by Induction**) (**25 points**).
A class of any size of 18 or more can be assembled from student teams of sizes 4 and 7. Prove this by **induction** (of some kind), using the induction hypothesis:

$$S(n) ::= \text{a class of } n + 18 \text{ students can be assembled from teams of sizes 4 and 7.}$$