IMPLIES

IMPLIES

The value of (P IMPLIES Q) is F iff P is T and Q is F.

Truth Table for IMPLIES

P	Q	P IMPLIES Q
T	T	T
T	F	F
F	T	T
F	F	T

A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to reach the false conclusion

A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to reach the false conclusion

A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to reach the false conclusion from the false hypothesis.

A True Implication

(1=-1) IMPLIES (I am Pope)
We reasoned *correctly* to reach the false conclusion from the false hypothesis.

A True Implication

(1=-1) IMPLIES (I am Pope)
The *whole implication* is true, even though both conclusion & hypothesis are false.

Proving Validity

Instead of truth tables, can try to prove valid formulas symbolically using axioms and deduction rules

modus ponens rule

$$\frac{\overbrace{P,\quad P\ \text{IMPLIES}\ Q}^{\text{antecedents}}}{\underbrace{Q}_{\text{conclusion}}}$$

Soundness

A sound rule preserves truth: if all the antecedents are true in some environment, then so is the conclusion.

Soundness

modus ponens is sound:
if P is true,
and P IMPLIES Q is true,
then Q must be true
—by truth table.

Soundness & Validity

Lemma: A rule is sound iff
AND{Antecedents}
IMPLIES Conclusion
is valid.