The value of \((P \implies Q)\) is \(F\) iff

- \(P\) is \(T\) and \(Q\) is \(F\).

**Truth Table for IMPLIES**

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>(P \implies Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

**A True Implication**

\((1=-1) \implies (I\ am\ Pope)\)

We reasoned correctly to reach the false conclusion.
A True Implication

\[(1=-1) \text{ IMPLIES (I am Pope)}\]

We reasoned correctly to reach the false conclusion from the false hypothesis.

The whole implication is true, even though both conclusion & hypothesis are false.

Proving Validity

Instead of truth tables, can try to prove valid formulas symbolically using axioms and deduction rules.
Soundness

**modus ponens** rule

antecedents

\[ P, \ P \ \text{IMPLIES} \ Q \]

conclusion

---

Soundness

A sound rule preserves truth: if all the antecedents are true in some environment, then so is the conclusion.

---

Soundness & Validity

Lemma: A rule is sound iff

\[ \text{AND\{Antecedents\}} \ \text{IMPLIES} \ \text{Conclusion} \]

is valid.