

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Proof by **Cases**: Friends & Strangers



Albert R Meyer

February 8, 2015

friend-strangers.1

6	9	13	7
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Friends & **Strangers**

Six people. Every two are either friends or strangers.

Claim: there is a set of
3 mutual friends or
3 mutual strangers



Albert R Meyer

February 8, 2015

friend-strangers.2

6	9	13	7
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Friends & **Strangers**

People are circles

3 mutual strangers

3 mutual friends



red line shows friends

blue line shows strangers



Albert R Meyer

February 8, 2015

friend-strangers.3

6	9	13	7
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Friends & **Strangers**

Take 3 minutes to find a counter-example

--or convince yourself there isn't any counterexample, that is, the **Claim** is true.



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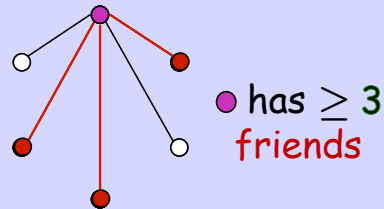
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friend-strangers.4

6	9	13	7
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A Proof of the Claim

- Person ● has a line to each of the 5 other people.
- lines are red or blue, so at least 3 must be the same color.



Albert R Meyer

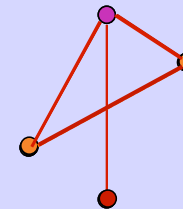
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friend-strangers.5

6	9	13	7
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A Proof of the Claim

Case 1: some pair of these friends are friends of each other, then we have 3 mutual friends:



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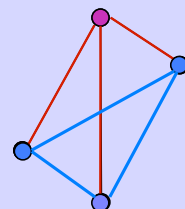
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friend-strangers.7

6	9	13	7
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A Proof of the Claim

Case 2: no pair of these friends are friends of each other, so we have 3 mutual strangers:



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friend-strangers.8

6	9	13	7
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A Proof of the Claim

Since the Claim is true in either case, and one of these cases always holds, the Claim is always true.

QED



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6	9	13	7
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Ramsey's Theorem

For any k , every *large enough* group of people will include either

k mutual friends, or
 k mutual strangers.



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friend-strangers.10

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Ramsey's Theorem

For any k , every *large enough* group of people will include either

size- k red clique, or
size- k blue clique.

Let $R(k)$ be the *large enough* size.

So we've proved that $R(3) = 6$.



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friend-strangers.11

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Ramsey's Numbers

Turns out that $R(4) = 18$ (not easy!)

$R(5)$ is unknown!

Paul Erdős considered finding $R(6)$
a hopeless challenge!

So in our second class, we have
reached a research frontier!



Albert R Meyer

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friend-strangers.12