

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Bounds on Deviation Chebyshev Bound



6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

Improving the Markov Bound

$$\Pr[|R - \mu| \geq x] \\ = \Pr[(R - \mu)^2 \geq x^2]$$

by Markov:

$$\leq \frac{E[(R - \mu)^2]}{x^2}$$

variance of R



6	9	13	7
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Chebyshev Bound

$$\Pr[|R - \mu| \geq x] \leq \frac{\text{Var}[R]}{x^2}$$

$$\text{Var}[R] ::= E[(R - \mu)^2]$$



6	9	13	7
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Variance of a Random Variable

$$\text{Var}[R] ::= E[(R - \mu)^2]$$

Variance is also called the
mean square error



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Chebyshev Bound

$$\Pr[|R - \mu| \geq x] \leq \frac{\text{Var}[R]}{x^2}$$

$$\sigma_R ::= \sqrt{\text{Var}[R]}$$

standard deviation



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chebyshev.5

6	9	13	7
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Standard Deviation of an RV

Standard deviation is also called the

root mean square error

$$\sigma_R ::= \sqrt{\text{Var}[R]}$$

standard deviation



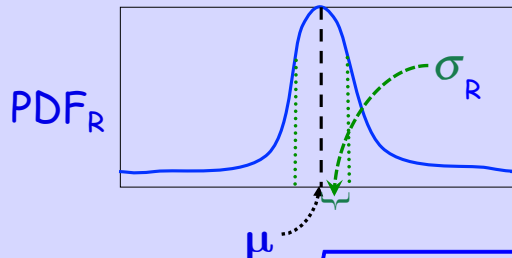
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6	9	13	7
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Standard Deviation of an RV



$$\sigma_R ::= \sqrt{\text{Var}[R]}$$



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6	9	13	7
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Chebyshev Bound

$$\Pr[|R - \mu| \geq x] \leq \frac{\sigma_R^2}{x^2}$$

$$\sigma_R ::= \sqrt{\text{Var}[R]}$$



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6	9	13	7
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Chebyshev Bound (Restated)

$$\Pr[|R - \mu| \geq c\sigma_R] \leq \frac{1}{c^2}$$

$$\sigma_R ::= \sqrt{\text{Var}[R]}$$



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6	9	13	7
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15	8	11	2

Standard Deviation

$$\Pr[|R - \mu| \geq c\sigma_R] \leq \frac{1}{c^2}$$

R probably not many σ 's from μ :
further than σ $\Pr \leq 1$

2σ $\Pr \leq 1/4$

3σ $\Pr \leq 1/9$

4σ $\Pr \leq 1/16$



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chebyshev.10