

In-Class Problems Week 9, Wed.

Problem 1.

Four Students want separate assignments to four VI-A Companies. Here are their preference rankings:

Student	Companies
Albert:	HP, Bellcore, AT&T, Draper
Sarah:	AT&T, Bellcore, Draper, HP
Tasha:	HP, Draper, AT&T, Bellcore
Elizabeth:	Draper, AT&T, Bellcore, HP

Company	Students
AT&T:	Elizabeth, Albert, Tasha, Sarah
Bellcore:	Tasha, Sarah, Albert, Elizabeth
HP:	Elizabeth, Tasha, Albert, Sarah
Draper:	Sarah, Elizabeth, Tasha, Albert

- (a) Use the Mating Ritual to find *two* stable assignments of Students to Companies.
- (b) Describe a simple procedure to determine whether any given stable marriage problem has a unique solution, that is, only one possible stable matching.

Problem 2.

Suppose that Harry is one of the boys and Alice is one of the girls in the *Mating Ritual*. Which of the properties below are preserved invariants? Why?

- Alice is the only girl on Harry's list.
- There is a girl who does not have any boys serenading her.
- If Alice is not on Harry's list, then Alice has a suitor that she prefers to Harry.
- Alice is crossed off Harry's list, and Harry prefers Alice to anyone he is serenading.
- If Alice is on Harry's list, then she prefers Harry to any suitor she has.

Problem 3.

A preserved invariant of the Mating Ritual is:

For every girl, G , and every boy, B , if G is crossed off B 's list, then G has a favorite suitor, and she prefers him over B .

Use the invariant to prove that the Mating Algorithm produces stable marriages. (Don't look up the proof in the Notes or slides.)

Problem 4.

The most famous application of stable matching was in assigning graduating medical students to hospital residencies. Each hospital has a preference ranking of students, and each student has a preference ranking of hospitals, but unlike finding stable marriages between an equal number of boys and girls, hospitals generally have differing numbers of available residencies, and the total number of residencies may not equal the number of graduating students.

Explain how to adapt the Stable Matching problem with an equal number of boys and girls to this more general situation. In particular, modify the definition of stable matching so it applies in this situation, and explain how to adapt the Mating Ritual to handle it.