

## In-Class Problems Week 6, Wed.

### Problem 1.

Find the remainder of  $26^{1818181}$  divided by 297.

*Hint:*  $1818181 = (180 \cdot 10101) + 1$ ; use Euler's theorem.

**Problem 2.** (a) Prove that  $2012^{1200}$  has a multiplicative inverse modulo 77.

(b) What is the value of  $\phi(77)$ , where  $\phi$  is Euler's function?

(c) What is the remainder of  $2012^{1200}$  divided by 77?

### Problem 3.

Prove that for any prime,  $p$ , and integer,  $k \geq 1$ ,

$$\phi(p^k) = p^k - p^{k-1},$$

where  $\phi$  is Euler's function. *Hint:* Which numbers between 0 and  $p^k - 1$  are divisible by  $p$ ? How many are there?

**Note:** This is proved in the text. Don't look up that proof.

### Problem 4.

At one time, the Guinness Book of World Records reported that the "greatest human calculator" was a guy who could compute 13th roots of 100-digit numbers that were 13th powers. What a curious choice of tasks...

In this problem, we prove

$$n^{13} \equiv n \pmod{10} \tag{1}$$

for all  $n$ .

(a) Explain why (1) does not follow immediately from Euler's Theorem.

(b) Prove that

$$d^{13} \equiv d \pmod{10} \tag{2}$$

for  $0 \leq d < 10$ .

(c) Now prove the congruence (1).