In-Class Problems Week 14, Wed.

Problem 1. **(a)** Find a stationary distribution for the random walk graph in Figure 1.

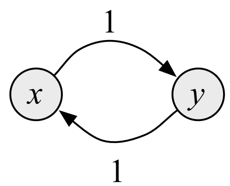

Figure 1

(b) Explain why a long random walk starting at node x in Figure 1 will not converge to a stationary distribution. Characterize which starting distributions will converge to the stationary one.

(c) Find a stationary distribution for the random walk graph in Figure 2.

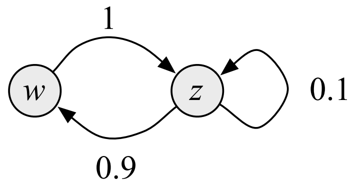

Figure 2

(d) If you start at node w Figure 2 and take a (long) random walk, does the distribution over nodes ever get close to the stationary distribution? You needn't prove anything here, just write out a few steps and see what's happening.

(e) Explain why the random walk graph in Figure 3 has an uncountable number of stationary distributions.

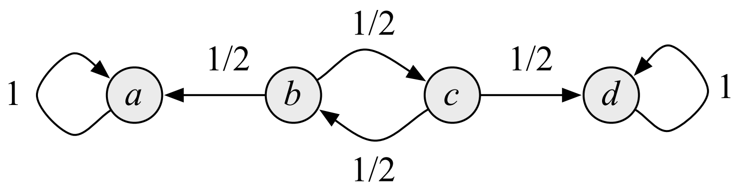

Figure 3

(f) If you start at node b in Figure 3 and take a long random walk, the probability you are at node d will be close to what fraction? Explain.

(g) Give an example of a random walk graph that is not strongly connected but has a unique stationary distribution. *Hint:* There is a trivial example.

Problem 2.
Prove that for finite random walk graphs, the uniform distribution is stationary if and only the probabilities of the edges coming into each vertex always sum to 1, namely

$$\sum_{u \in \text{into}(v)} p(u, v) = 1, \tag{1}$$

where $\text{into}(w) ::= \{v \mid \langle v \to w \rangle \text{ is an edge}\}$.

Problem 3.
A Google-graph is a random-walk graph such that every edge leaving any given vertex has the same probability. That is, the probability of each edge $\langle v \to w \rangle$ is $1/\operatorname{outdeg}(v)$.

A digraph is *symmetric* if, whenever $\langle v \to w \rangle$ is an edge, so is $\langle w \to v \rangle$. Given any finite, symmetric Google-graph, let

$$d(v) ::= \frac{\operatorname{outdeg}(v)}{e},$$

where e is the total number of edges in the graph.

(a) If d was used for webpage ranking, how could you hack this to give your page a high rank? ...and explain informally why this wouldn't work for "real" page rank using digraphs?

(b) Show that d is a stationary distribution.