

In-Class Problems Week 12, Wed.

Problem 1.

There is an unpleasant degenerative disease called Beaver Fever which causes people to tell unrelenting math jokes in social settings, believing other people would think they're funny. Fortunately, Beaver Fever is rare, afflicting only about 1 in 1000 people. Doctor Meyer has a fairly reliable diagnostic test to determine who is going to suffer from this disease:

- If a person will suffer from Beaver Fever, the probability that Dr. Meyer diagnoses this is 0.99.
- If a person will not suffer from Beaver Fever, the probability that Dr. Meyer diagnoses this is 0.97.

Let B be the event that a randomly chosen person will suffer Beaver Fever, and Y be the event that Dr. Meyer's diagnosis is "Yes, this person will suffer from Beaver Fever," with \overline{B} and \overline{Y} being the complements of these events.

(a) The description above explicitly gives the values of the following quantities. What are their values?

$$\Pr[B] \quad \Pr[Y \mid B] \quad \Pr[\overline{Y} \mid \overline{B}]$$

(b) Write formulas for $\Pr[\overline{B}]$ and $\Pr[Y \mid \overline{B}]$ solely in terms of the explicitly given quantities in part (a)—literally use their expressions, not their numeric values.

(c) Write a formula for the probability that Dr. Meyer says a person will suffer from Beaver Fever solely in terms of $\Pr[B]$, $\Pr[\overline{B}]$, $\Pr[Y \mid B]$ and $\Pr[Y \mid \overline{B}]$.

(d) Write a formula solely in terms of the expressions given in part (a) for the probability that a person will suffer Beaver Fever given that Doctor Meyer says they will. Then calculate the numerical value of the formula.

Suppose there was a vaccine to prevent Beaver Fever, but the vaccine was expensive or slightly risky itself. If you were sure you were going to suffer from Beaver Fever, getting vaccinated would be worthwhile, but by part (d), even if Dr. Meyer diagnosed you as a future sufferer of Beaver Fever, the probability you actually will suffer Beaver Fever remains low (less than 1/30).

In this case, you might sensibly decide not to be vaccinated (after all, Beaver Fever is not *that* bad an affliction). So the diagnostic test serves no purpose in your case—you may as well not have bothered to get diagnosed. Even so, the test may be useful:

(e) Suppose Dr. Meyer had enough vaccine to treat 2% of the population. If he randomly chose people to vaccinate, he could expect to vaccinate only 2% of the people who needed it. But by testing everyone and only vaccinating those diagnosed as future sufferers, he can expect to vaccinate a much larger fraction of people who were going to suffer from Beaver Fever. Estimate this fraction.

Problem 2.

There are three prisoners in a maximum-security prison for fictional villains: the Evil Wizard Voldemort, the Dark Lord Sauron, and Little Bunny Foo-Foo. The parole board has declared that it will release two of the three, chosen uniformly at random, but has not yet released their names. Naturally, Sauron figures that he will be released to his home in Mordor, where the shadows lie, with probability $2/3$.

A guard offers to tell Sauron the name of one of the other prisoners who will be released (either Voldemort or Foo-Foo). If the guard has a choice of naming either Voldemort or Foo-Foo (because both are to be released), he names one of the two with equal probability.

Sauron knows the guard to be a truthful fellow. However, Sauron declines this offer. He reasons that knowing what the guard says will reduce his chances, so he is better off not knowing. For example, if the guard says, "Little Bunny Foo-Foo will be released", then his own probability of release will drop to $1/2$ because he will then know that either he or Voldemort will also be released, and these two events are equally likely.

Dark Lord Sauron has made a typical mistake when reasoning about conditional probability. Using a tree diagram and the four-step method, **explain his mistake**. What is the probability that Sauron is released given that the guard says Foo-Foo is released?

Hint: Define the events S , F , and " F " as follows:

$"F"$ = Guard says Foo-Foo is released

F = Foo-Foo is released

S = Sauron is released

Problem 3.

There are two decks of cards. One is complete, but the other is missing the Ace of spades. Suppose you pick one of the two decks with equal probability and then select a card from that deck uniformly at random. What is the probability that you picked the complete deck, given that you selected the eight of hearts? Use the four-step method and a tree diagram.

Supplemental problem**Problem 4.**

Suppose you repeatedly flip a fair coin until you see the sequence HTT or HHT. What is the probability you see the sequence HTT first?

Hint: Try to find the probability that HHT comes before HTT conditioning on whether you first toss an H or a T. The answer is not $1/2$.