

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
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Proof by Contradiction



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contradiction.1

6	9	13	7
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Proof by Contradiction

Is $\sqrt[3]{1332} \leq 11$?

If so, $1332 \leq 1331$

That's **not true**, so

$\sqrt[3]{1332} > 11$



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contradiction.2

6	9	13	7
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Proof by Contradiction

If an assertion implies something **false**, then the assertion itself must be **false**!



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contradiction.3

6	9	13	7
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Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

- Suppose $\sqrt{2}$ was **rational**
- So have n, d integers **without common prime factors** such that $\sqrt{2} = \frac{n}{d}$
- We will show that n & d are **both even**. This **contradicts no common factor**.



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contradiction.4

6	9	13	7
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Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

so can assume

$$\sqrt{2} = \frac{n}{d}$$

$$\sqrt{2}d = n$$

$$2d^2 = n^2$$

So n is **even**

QED

$$n = 2k$$

$$n^2 = 4k^2$$

$$2d^2 = 4k^2$$

$$d^2 = 2k^2$$

So d is **even**



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contradiction.5

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Quickie

Proof assumes that if n^2 is even, then n is even.

Why is this true?



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contradiction.6