Launchings
by David Bressoud

MONDAY, AUGUST 1, 2011

The Best Way to Learn

> *"The best way to learn is to do; the worst way to teach is to talk." — Paul Halmos* [1]

Last month, in *The Worst Way to Teach* [2], I wrote about some of the problems with instruction delivered by lecture. It stirred up a fair amount of discussion. Richard Hake started a thread on the MathForum [3]. He added several references to my own list and sparked a discussion that produced some heat and a lot of light. I do want to clarify that I recognize how important what I say in the classroom can be, as I will expound a bit later in this column. Nevertheless, I stand by my statement that "sitting still, listening to someone talk, and attempting to transcribe what they have said into a notebook is a very poor substitute for actively engaging with the material at hand, for doing mathematics."

I also want to respond to a number of people who stated that reliance solely on lecture is not the real problem with mathematics instruction today; the real problem is ... It was not my claim that moving away from pure lecture would solve all of our problems or even our greatest problems in mathematics instruction, merely that there are better ways to teach.

One high school teacher asked me for practical suggestions of things he could do to more actively engage his students. Fortunately, there are a number of resources. The one I pointed him to and that I want to talk more about in this column is the Academy of Inquiry Based Learning, a clearinghouse of information about and resources for Inquiry Based Learning (IBL) [4].

What is IBL?

IBL is a descendant of the method of instruction made famous by R.L. Moore at the University of Texas at Austin. Moore would give his students basic definitions together with statements of theorems that used those terms. Students were forbidden to draw on any sources other than own intellect to prove the theorems. Class time was spent entirely in student presentation of proofs, which would be critiqued by the class. It was a demanding regimen that produced many research mathematicians and six presidents of the MAA: R.H. Bing, R.L. Wilder, E.E. Moise, G.S. Young, Jr., Richard Anderson, and Lida Barret were all students of R.L. Moore.

The pure Moore Method was taught in small graduate classes with hand picked students, but Moore also adapted his method for teaching calculus, and many others since have modified his approach to fit the needs of their own students. The core of what we have learned from Moore is that the teacher needs to talk less and the students need to do more. This is the essence of IBL. On the website of the Academy of Inquiry Based Learning, IBL is described as follows:

> *Boiled down to its essence IBL is a teaching method that engages students in sense-making activities. Students are given tasks requiring them to solve problems, conjecture, experiment, explore, create, and communicate... all those wonderful skills and habits of mind that Mathematicians engage in regularly. Rather than showing facts or a clear, smooth path to a solution, the instructor guides students via well-crafted problems through an adventure in mathematical discovery.* [5]

As this quote indicates, there is a very "big tent" approach to IBL today. Each year, the Educational Advancement Foundation sponsors a *Legacy of R.L. Moore* conference. In addition to the enthusiasm of the several hundred participants, what I find most impressive is the variety of ways in which people plug into the basic idea of IBL. At the

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latest conference, held in Washington, DC, June 2–4, Ted Mahavier talked about how to just get started by using one day per week for students to go to the board to present their homework solutions, Eric Hsu explained the connections to Triesman's *Emerging Scholars Program*, Angie Hodge showed how this approach is expressed in the Math Teachers' Circles, and Tom Banchoff explained his own take on IBL, which uses his course management software to enable his students to critique and learn from each others' proofs.

My own IBL experience

Ever since I taught calculus at Penn State in 1993–94 with David Smith and Lang Moore's *Project CALC* materials, I have recognized the importance of using at least some of my class time to engage students in the creative activity of doing mathematics: exploring, conjecturing, proving, and—most important—communicating. But such activities always were in conjunction with a fair dose of my own explanation of what is important and how to think about the mathematics. I am not prepared to give up that role. It is an important part of what it means for me to teach. But I am learning how to cede more of my control over what happens in the classroom.

This past year, I taught our junior/senior Number Theory course using *Number Theory through Inquiry* by Marshall, Odell and Starbird [6]. The great advantage of this book is that it presents all of the theorems of elementary number theory but none of the proofs. As I used this book, I talked more than others might. Students were required to read and think about the theorems before we met in class. Class would start by answering questions they had about the reading, followed by student presentations of the results they were able to prove. Most of the class was spent discussing the more challenging proofs. Here, when the silence stretched too long, I would step in and explain how to think about this proof, perhaps even sketch a possible outline. For each class period, I identified several key proofs that each student would be required to write up and submit as homework. I did write out complete proofs in the first class or two, to explain the difference between the sequence of personal insights that convinces oneself that a complete proof has been found and the way one writes up a proof for public consumption. After that, I never again wrote out a complete proof. That was their responsibility, and that was the basis for their grade.

Unlike Moore, I encouraged students to work together and critique each other's proofs outside of class. But each student had to produce his or her own complete written proof of each of the important theorems. At the end of the semester, I was pleasantly surprised at how much the students appreciated this experience. They greatly preferred creating their own proofs over trying to learn from someone else's.

IBL: The study

Over the past several years, the Educational Advancement Foundation has sponsored a study of the effectiveness of IBL at four universities with established centers for the support of IBL courses: University of California, Santa Barbara; University of Texas, Austin; University of Michigan, Ann Arbor; and the University of Chicago. Sandra Laursen and her team of sociologists at the University of Colorado, Boulder conducted the study [7].

This was a large, complex undertaking that was complicated by the fact IBL was being implemented in a wide variety of types of courses, from mathematics for prospective elementary teachers through upper division mathematics, and in only a few cases were there comparable IBL and non-IBL sections. Nevertheless, there was a real difference in the way IBL and non-IBL courses were taught. On average, student-centered activities made up over 60% of class time in IBL courses. In non-IBL classes, the instructor talked an average of 87% of the time.

What Laursen found was that IBL made a difference across many areas. It produced higher cognitive gains, including understanding of mathematical concepts and improved thinking and problem-solving skills; higher affective gains, including increased confidence, improved attitude, and greater persistence; and higher social gains, including ability to collaborate and explain mathematical ideas to others. Laursen also found that the percentage of time that the instructor spent on student-centered activities was the single best predictor of student gains.

The strongest gains were observed among women and students with weak prior achievement. These gains appeared not just in the IBL class but also continued through subsequent required mathematics courses, whether or not they were taught using IBL. This happened without decreasing the achievement levels of men and students with strong prior achievement.

The fact that traditionally underrepresented groups of students benefit most from IBL should not be surprising. Those of us now teaching in our colleges and universities succeeded in the existing system because we knew or managed to learn how to convert the lectures into active engagement with the mathematics. Lecturing worked for us. But it does not work for the many students who have never learned how to study mathematics. Perhaps the best news from this study is that pulling time away from lecture does nothing to decrease the learning of those who best know how to benefit from that style of teaching. There is hope that by changing how we teach we can increase the population of students who can do mathematics.

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[1] P. R. Halmos, E. E. Moise, and George Piranian. May, 1975. The Problem of Learning to Teach. *The American Mathematical Monthly*. Vol. 82, no. 5, 466–476.

[2]Bressoud, D. July, 2011. The Worst Way to Teach, *Launchings*

[3]Hake, R. Re: Lecture Isn't Effective: More Evidence. *The Math Forum @ Drexel*. July 15, 2011 12:56 PM

[4] Academy of Inquiry Based Learning.

[5] *What is IBL?* Academy of Inquiry Based Learning.

[6] Marshall, D.C., E. Odell, M. Starbird. 2007. *Number Theory through Inquiry*. The Mathematical Association of America. Washington, D.C.

[7] Laursen, S., M.L. Hassi, M. Kogan, A.-B. Hunter, T. Weston. 2011. *Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics*. University of Colorado, Boulder.

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Posted by Mathematical Association of America at 12:00 AM

2 comments:

R. Wright said...

Lecturing worked for us.

I have to take issue with that statement. I think it would be more accurate to say "Lecturing did not keep us from learning." If anything, I would say for myself that in many mathematics classes I learned **despite** the lecturing.

August 1, 2011 7:01 PM

Daniel Asimov said...

It is true that little learning takes place without doing. But I find no reason that a large portion of class time need be taken up with the students' doing math rather than actively listening to a lecture. Particularly since modern college math curricula can barely fit into the time allotted for lectures during the term.

Some of the class time indeed should require the students' doing. I will often ask questions of my students during a lecture, and if no one has any idea, I'll let them spend some time thinking about it.

But doing is exactly what homework is for. And some class time --whether it be in the traditional lecture section or in the recitation section (if any) -- must be taken up with explaining how to do at least a sampling of the homework problems.

I *do* heartily agree with Dr. Bressoud that nothing is gained by making the students take dictation from the lecturer in order for them to have a record of the lecture. Ideally the students will be given lecture notes, obviating the need for this task and freeing up the students to pay attention and try to understand the lecturer in real time.

August 24, 2011 5:39 PM

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