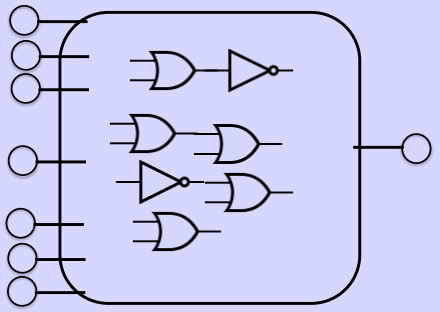
Mathematics for Computer Science

MIT 6.042J/18.062J

SAT Reduces to 3-Coloring

Circuit SAT



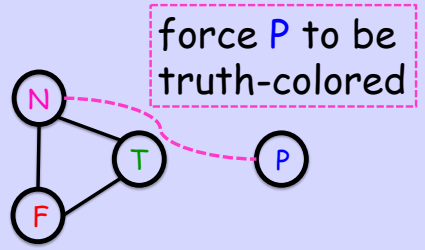
Is there an assignment of T's and F's to the inputs that yields output T?

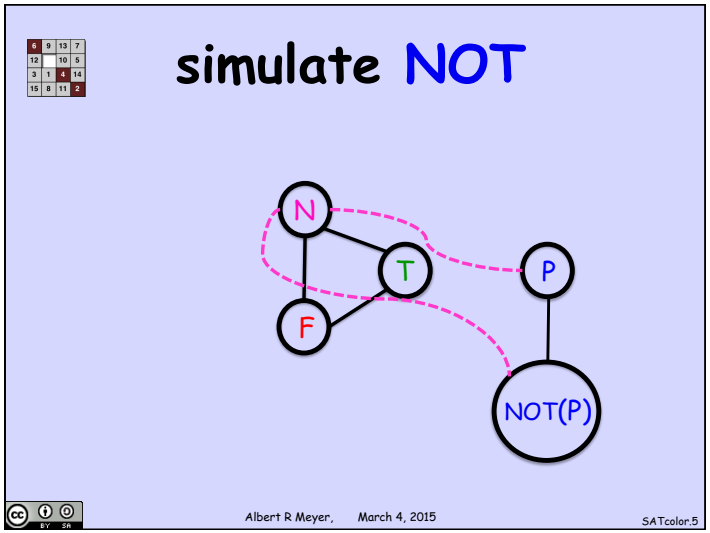
Circuit SAT

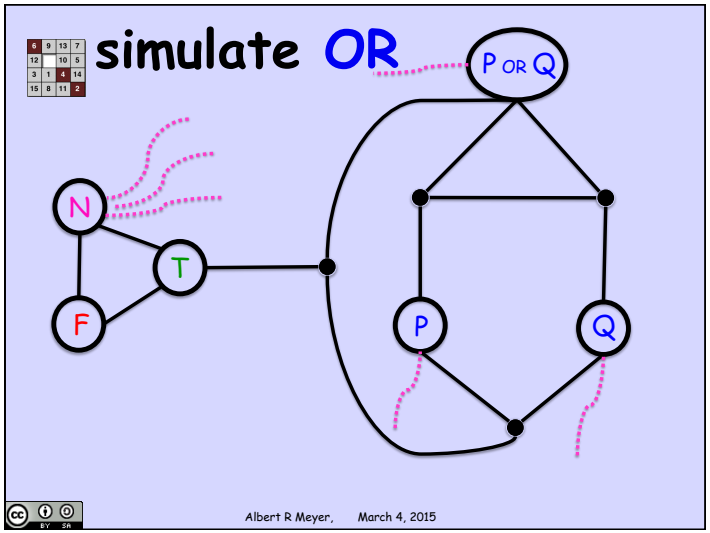
Create graph whose 3-colorings simulate circuit behavior

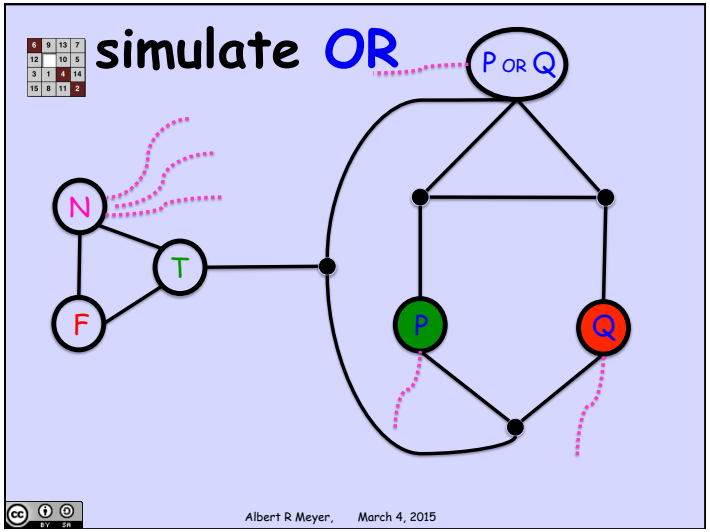
Is there an assignment of T's and F's to the inputs that yields output T?

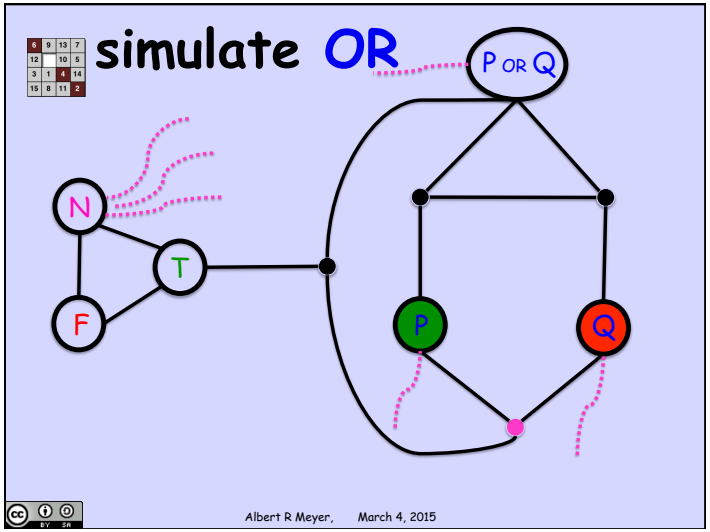
Truth Colors

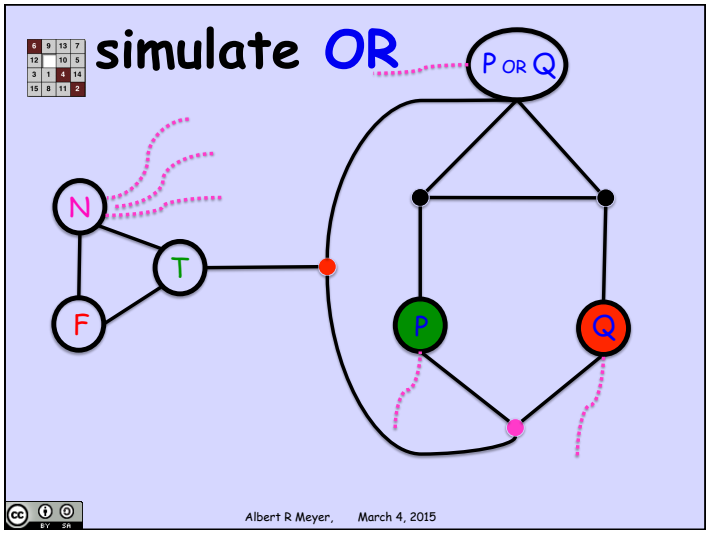


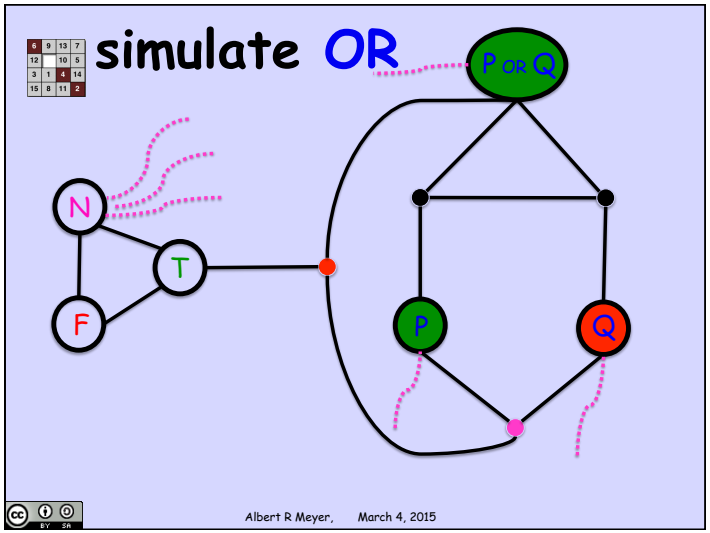

simulate NOT
N
T
F
P
NOT(P)
Albert R Meyer, March 4, 2015
SATcolor.5

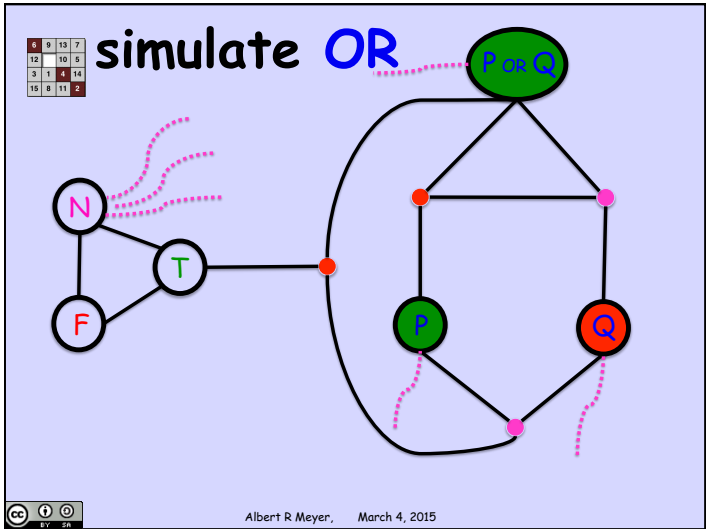

simulate OR
P OR Q
N
T
F
P
Q
Albert R Meyer, March 4, 2015

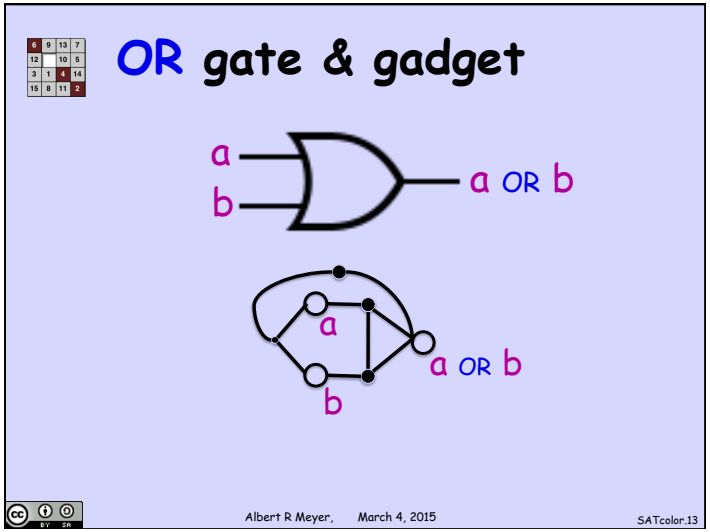

simulate OR
P OR Q
N
T
F
P
Q
Albert R Meyer, March 4, 2015


simulate OR
P OR Q
N
T
F
P
Q
Albert R Meyer, March 4, 2015

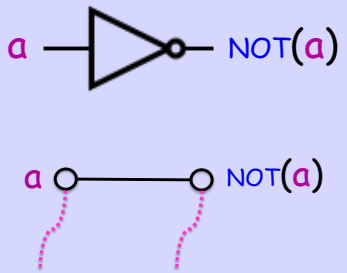

simulate OR
P OR Q
N
T
F
P
Q
Albert R Meyer, March 4, 2015


simulate OR
P OR Q
N
T
F
P
Q
Albert R Meyer, March 4, 2015

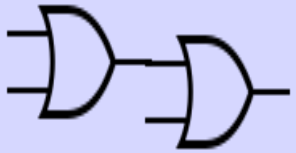

simulate OR
P OR Q
N
T
F
P
Q
Albert R Meyer, March 4, 2015


OR gate & gadget
a
b
a OR b
a
b
a OR b
Albert R Meyer, March 4, 2015
SATcolor.13

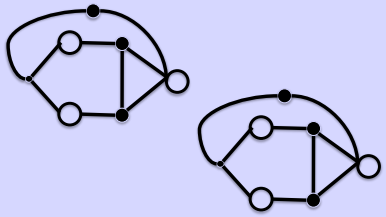
NOT gate & gadget



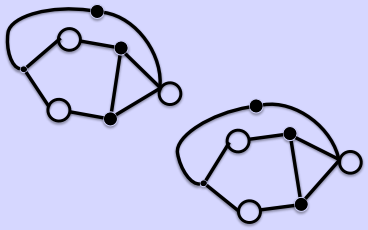
Circuit SAT

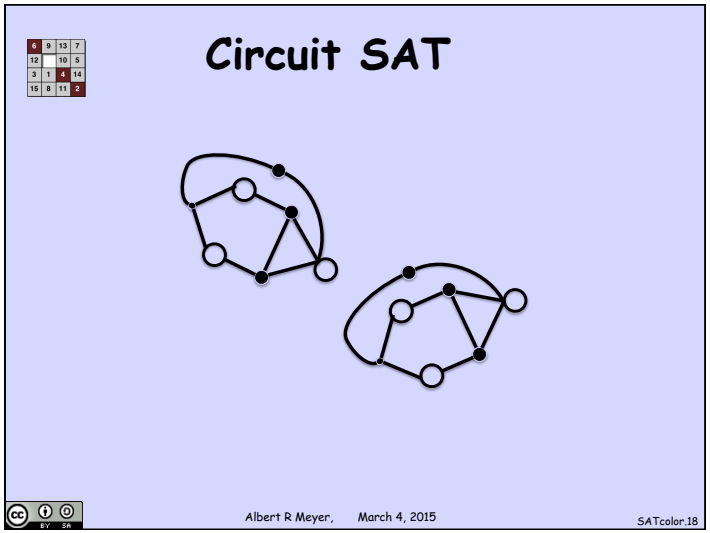


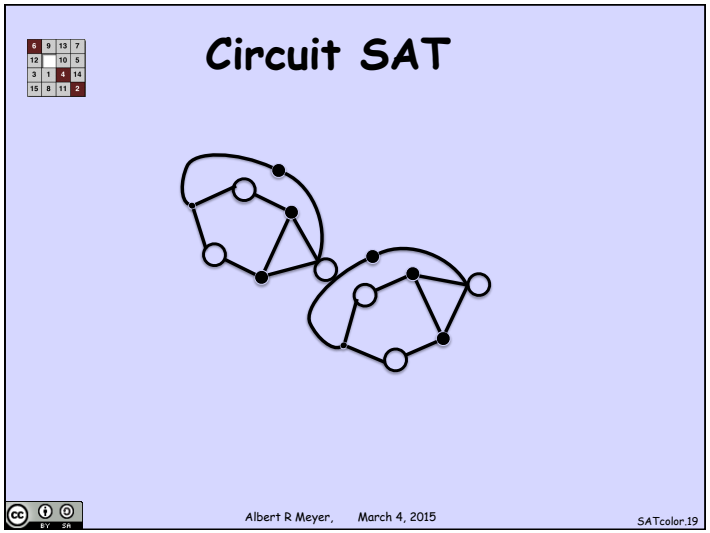
Circuit SAT

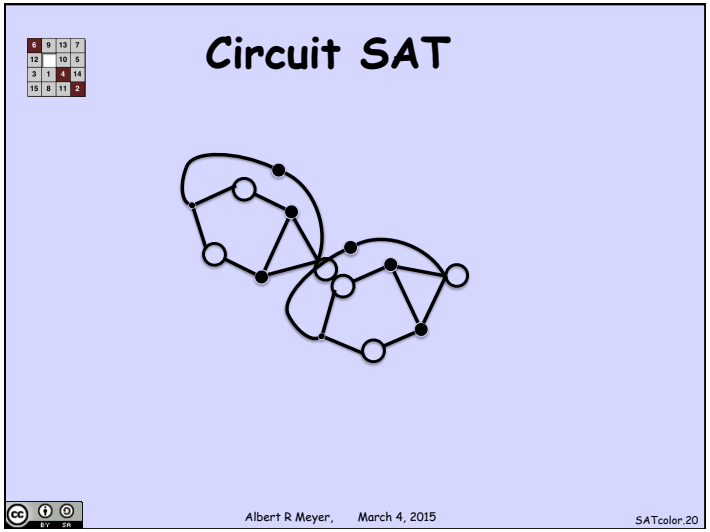


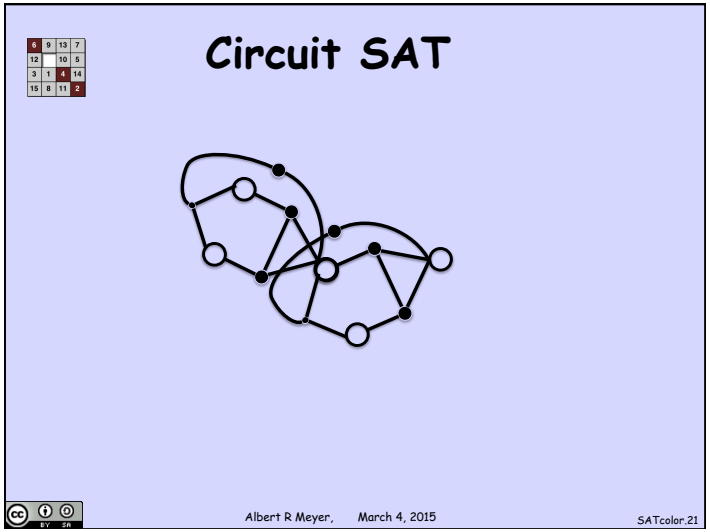
Circuit SAT



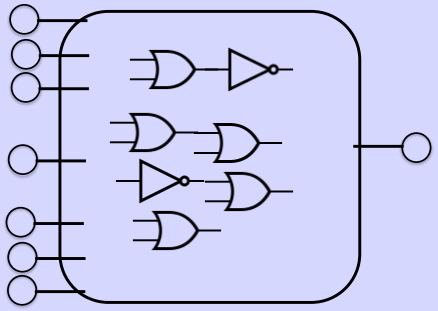

Circuit SAT
Albert R Meyer, March 4, 2015
SATcolor.18


Circuit SAT
Albert R Meyer, March 4, 2015
SATcolor.19

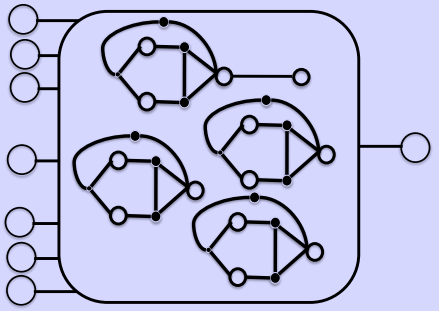

Circuit SAT
Albert R Meyer, March 4, 2015
SATcolor.20


Circuit SAT
Albert R Meyer, March 4, 2015
SATcolor.21

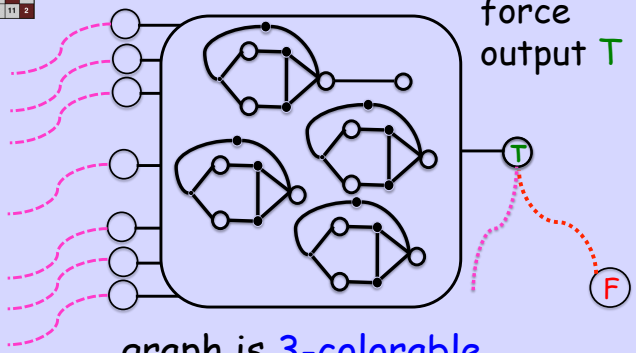
Circuit SAT



Circuit SAT



Circuit SAT



graph is 3-colorable
iff circuit is in SAT

SAT vs 3-Color

SAT and 3-color stand and fall together

SAT vs 3-Color

SAT and 3-color stand and fall together: there is an "efficient" (polynomial time) SAT procedure iff there is one for 3-color.

SAT vs 3-Color

SAT and 3-color stand and fall together: there is an "efficient" (polynomial time) SAT procedure iff there is one for 3-color. Both problems are NP-complete.