The Well Ordering Principle, III

Geometric sums

\[ 1 + r + r^2 + r^3 + \ldots + r^n = \frac{r^{n+1} - 1}{r - 1} \]

Proof by WOP. Let \( m \) be smallest \( n \) with \( \neq \). But \( = \) for \( n = 0 \), so \( m > 0 \), and

\[ 1 + r + r^2 + r^3 + \ldots + r^{m-1} = \frac{r^{m} - 1}{r - 1} \]

add \( r^m \) to both sides

LHS = \( 1 + r + r^2 + r^3 + \ldots + r^{m-1} + r^m \)

RHS = \( \frac{r^m - 1}{r - 1} + \frac{r^{m+1} - r^m}{r - 1} = \frac{r^{m+1} - 1}{r - 1} \)

so = at \( m \), contradicting \( \neq \): there is no counterexample.

Well Ordering Principle Proofs

To prove \( \forall n \in \mathbb{N}. P(n) \) using WOP:

- define set of counterexamples
  \[ C := \{ n \in \mathbb{N} \mid \text{NOT } P(n) \} \]
- assume \( C \) is not empty. By WOP, have minimum element \( m \in C \)
- Reach a contradiction somehow ...

...or by proving \( P(m) \)