The Well Ordering Principle, II

Prime Products

Thm: Every integer \( > 1 \) is a product of primes.

Proof: (by contradiction) Suppose \{nonproducts\} is nonempty. By WOP, there is a least \( m > 1 \) that is a nonproduct. This \( m \) is not prime (else is a product of 1 prime).

Prime Products

Thm: Every integer \( > 1 \) is a product of primes.

...So \( m = j \cdot k \) for integers \( j,k \)
where \( m > j,k > 1 \). Now \( j,k < m \)
so both are prime products:
\( j = p_1 \cdot p_2 \cdots p_{94} \)
\( k = q_1 \cdot q_2 \cdots q_{213} \)

So \{counterexamples\} = \( \emptyset \). QED

Well Ordered Postage

available stamps: 5¢ 3¢

n is postal if can make \((n+8)\)¢ postage from 3¢ & 5¢ stamps.

Well Ordered Postage

available stamps: 5¢ 3¢

Thm: Every number is postal.
Prove by WOP. Suppose not. Let \( m \) be least counterexample.
That is,
• \( m \) is not postal,
• any number \( < m \) is postal

0 is postal: so \( m \neq 0 \)

\[ m \neq 1: \quad m \neq 2: \]

Hence, \( m \geq 3 \).

Now \( m-3 \) is a number \( < m \), so is postal. But then \( m \) is postal too:
\[ (m-3) + 3\text{¢} + 8\text{¢} = m + 8\text{¢} \]
\[ \text{contradiction!} \]