What is a Set?

Informally:
A set is a collection of mathematical objects, with the collection treated as a single mathematical object.
(This is circular of course: what’s a collection?)

Familiar sets

- real numbers \( \mathbb{R} \)
- complex numbers \( \mathbb{C} \)
- integers \( \mathbb{Z} \)
- empty set \( \emptyset \)

A set of 4 things

\{7, “Albert R.”, \(\pi/2\), \(T\)\}

A set with 4 elements: two numbers, a string, and a Boolean.
Same as
\{\(T\), “Albert R.”, 7, \(\pi/2\)\}
-- order doesn’t matter
In or Not In

An element is in or not in a set:
\{7, \pi/2, 7\} is same as \{7, \pi/2\}
No notion of being in the set more than once.

Membership

\( x \) is a member of \( A \): \( x \in A \)
\( \pi/2 \in \{7, “Albert R.”, \pi/2, T\} \)
\( 14/2 \in \)
\( \pi/3 \notin \)

Synonyms for Membership

\( x \in A \)
\( x \) is an element of \( A \)
\( x \) is in \( A \)

examples:
\( 7 \in \mathbb{Z}, \ 2/3 \notin \mathbb{Z}, \ \mathbb{Z} \in \{T, \mathbb{Z}, 7 \} \)

Subset (\( \subseteq \))

\( A \subseteq B \) \( A \) is a subset of \( B \)
\( A \) is contained in \( B \)
Every element of \( A \) is also an element of \( B \):
\( \forall x \ [x \in A \ IMPLIES \ x \in B] \)
Subset

examples:

\( \mathbb{Z} \subseteq \mathbb{R} \), \( \mathbb{R} \subseteq \mathbb{C} \), \( \{3\} \subseteq \{5,7,3\} \)

\( A \subseteq A \), \( \emptyset \subseteq \) every set

\( \emptyset \subseteq \) everything

\( \emptyset \subseteq B \) is defined to mean

\( \forall x [x \in \emptyset \implies x \in B] \)

false \underline{true}

Defining Sets

The set of elements \( x \) in \( A \)

such that \( P(x) \) is true.

\( \{ x \in A \mid P(x) \} \)

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The set $E$ of even integers:

\[ \{ n \in \mathbb{Z} \mid n \text{ is even} \} \]

Power Set

\[ \text{pow}(A) ::= \text{all the subsets of } A = \{ B \mid B \subseteq A \} \]

example:

\[ \text{pow}\{\{T, F\}\} = \{ \{T\}, \{F\}, \{T, F\}, \emptyset \} \]

$E \in \text{pow}(\mathbb{Z})$, $\mathbb{Z} \in \text{pow}(\mathbb{R})$

\[ B \in \text{pow}(A) \text{ IFF } B \subseteq A \]