**Relations & Functions**

A binary relation associates elements of one set called the **domain**, with elements of another set called the **codomain**.

**“Registered for” relation** \( R \)

- Jason
- Joan
- Yihui
- Adam

**stuDent**

**subject**

\[ R(\text{Jason}, 6.042) \in R \]

\[ (\text{Jason}, 6.042) \in \text{graph}(R) \]

**Images under** \( R \)

\[ R(\text{Jason}) = \text{subjects Jason is registered for} \]
"Registered for" relation $R$

- $\text{student}$
  - Jason
  - Joan
  - Yihui
  - Adam

$R$ (Jason) = subjects Jason is registered for

- $6.042$
- $6.003$
- $6.012$
- $6.004$

$R(\{\text{Jason, Yihui}\})$

- subjects with Jason or Yihui registered

$R(\{\text{Jason, Yihui}\})$

- $6.042$
- $6.003$
- $6.012$
- $6.004$

Images under $R$

- $R(\{\text{Jason, Yihui}\})$ = subjects with Jason or Yihui registered
- $R(\{\text{Jason, Yihui}\})$ = subjects Jason is registered for
- $R(\{\text{Jason, Yihui}\})$ = all the subjects being taken by students in the set $X$
- $R(\{\text{Jason, Yihui}\})$ = everything $R$ relates to things in $X$
Images under $R$

$R(\{\text{Jason}, \text{Yihui}\}) = \{\text{subjects with Jason or Yihui registered} = \{6.042, 6.012, 6.004\}$

Images under $R$

$R(X) ::= \text{endpoints of arrows from points in } X$

$\{j \in J \mid \exists d \in X. d R j\}$

an arrow from $X$ goes to $j$

"registers" relation $R^{-1}$

```
Student
  Jason ⊳ registered for 6.042
  Joan ⊳ 6.003
  Yihui ⊳ 6.012
  Adam ⊳ 6.004
```

"registers" relation $R^{-1}$

```
d R j \iff j R^{-1} d
```

Images under $R^{-1}$

$R^{-1}(6.012) =$

```
Student
  Jason ⊲ registers 6.042
  Joan ⊲ 6.003
  Yihui ⊲ 6.012
  Adam ⊲ 6.004
```
Images under $R^{-1}$

$R^{-1}(6.012) = \{\text{Jason, Yihui}\}$

$R^{-1}(\{6.012, 6.003\}) = \{\text{Jason, Joan, Yihui}\}$

“registers” relation $R^{-1}$

```
<table>
<thead>
<tr>
<th>Student</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jason</td>
<td>6.042</td>
</tr>
<tr>
<td>Joan</td>
<td>6.003</td>
</tr>
<tr>
<td>Yihui</td>
<td>6.012</td>
</tr>
<tr>
<td>Adam</td>
<td>6.004</td>
</tr>
</tbody>
</table>
```

Inverse image under $R$

$R^{-1}(J) = \text{all the students registered for some subject}$

Every student is registered for some subject:

$D \subseteq R^{-1}(J)$

(not true: Adam wasn’t registered)

“advises” relation $V$

```
<table>
<thead>
<tr>
<th>Professor</th>
<th>Advises</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARM</td>
<td>FTL</td>
<td>Jason</td>
</tr>
<tr>
<td></td>
<td>TLP</td>
<td>Joan</td>
</tr>
<tr>
<td></td>
<td>LPK</td>
<td>Yihui</td>
</tr>
<tr>
<td></td>
<td>PHW</td>
<td>Adam</td>
</tr>
</tbody>
</table>
```
Composing \( R \) and \( V \)

\[
R(V(\{FTL, TLP\})) = R(\{Joan, Yihui, Adam\}) = \{6.003, 6.012, 6.004\}
\]

\( R(V(X)) \) denotes subjects that advisees of profs in \( X \) are registered for.

Composing \( R \) and \( V \)

\[
(R \circ V)(X) := R(V(X))
\]

\( R \circ V \) is the composition of \( R \) and \( V \).

Composing \( R \) and \( V \)

\[
R \circ V := \text{"prof has advisee registered for"}
\]

\( p(R \circ V)j := \text{prof } p \text{ has an advisee registered in subject } j \)
Composing \( R \) and \( V \)

\[
\text{ARM} \ (R \circ V) \ 6.012 \quad \text{because} \\
\text{ARM} \ V \ Yihui \quad \text{AND} \quad Yihui \ R \ 6.012 \\
p(R \circ V)_j \quad \text{IFF} \\
\exists d \in D. [pVd \quad \text{AND} \quad d \ R \ j] \\
\text{note:} \ V,R \ \text{in reverse order}
\]

"teaches" relation \( T \)

Profs should not teach their advisees:

\[
\forall p \ \forall j. \ \neg (p(R \circ V)_j \ \text{AND} \ \ pTj) \\
T \cap (R \circ V) = \emptyset
\]

\[
R \circ V \subseteq T
\]
A binary relation, $R$, from a set $A$ to a set $B$ associates elements of $A$ with elements of $B$.

$\text{graph}(R) := \{ (a_1, b_2), (a_1, b_4), (a_3, b_4) \}$

$\text{range}(R) = \{ b_2, b_4 \}$
A function, $F$, from $A$ to $B$ is a relation which associates each element, $a$, of $A$ with at most one element of $B$, called $F(a)$.

Functions are relations

A function $F:A \rightarrow B$ is a function IFF $\left| F(a) \right| \leq 1$

IFF

$\left[ a F b \text{ AND } a F b' \right]$ IMPLIES $b = b'$