Sound Proofs
Instead of truth tables, can try to **prove** valid formulas symbolically.
Instead of truth tables, can try to prove valid formulas symbolically using axioms and deduction rules.
modus ponens rule

antecedents

\[ P, \ P \ IMPLIES \ Q \]

Q

conclusion
Sound Rules

preserve validity:
if all the antecedents are valid
Sound Rules

preserve validity:
if all the antecedents are valid, then conclusion is valid.
A Sound Rule

modus ponens is sound:

if \( P \) is valid,
and \( P \ IMPLIES Q \) is valid,
then \( Q \) must be valid
—by truth table.
Sound Proofs

If axioms are valid, and proof rules are sound, then provable formulas are valid.
Strongly Sound Rules

preserve truth:
if all the antecedents are true in some environment, then so is the conclusion.
Strongly Sound Rules

Rule is strongly sound:

\[ \text{AND}\{\text{Antecedents}\} \implies \text{Conclusion} \]

is valid.
A Strongly Sound Rule

modus ponens

if $P$ is $T$ in assignment $A$, and

$P$ IMPLIES $Q$ is $T$ in $A$, then $Q$ is $T$ in $A$. 
Strongly Sound Proofs

If proof rules are strongly sound, then

\[ \text{AND\{Axioms\}} \implies \text{Conclusion} \]

is valid.