Number Theory: Die Hard

Generalized Die Hard

Under Die Hard rules, gal.’s in each bucket are linear combinations of $a$ and $b$

Did it with buckets:
3 gal. & 5 gal.
3 gal. & 9 gal.
Now $a$ gal. & $b$ gal.?
Claim: Can get any linear combination of \( a, b \) into a bucket (if there’s room for it). Namely, say \( 0 \leq sa + tb < b \). Get \( sa + tb \) into the \( b \) gal. bucket as follows:

**Generalized Die Hard**

In fact, no need to count: fill bucket \( a \), pour into \( b \) — if \( b \) fills, empty it — until desired gal.’s in \( b \)!

assume \( s > 0 \). do \( s \) times:

fill bucket \( a \), pour into \( b \) — if \( b \) fills, empty it.

total fills = \( sa \)

\( 0 \leq \) amount left < \( b \)

\# \( b \) emptyings must be \(-t\)