Staff Solutions to Mini-Quiz 6

STAFF NOTE: Simple Graphs: Degrees, Isomorphism, Matchings, Mating Ritual, Coloring, Connectivity, Ch.11.1–11.10

Problem 1 (4 points).
The bipartite graph \( H \) in Figure 1 has an easily checked property that ensures it has a matching of all the vertices in \( L(H) \) with vertices in \( R(H) \). Verify that \( H \) has this property—you do not need to find the matching.

\[ \begin{array}{c}
\begin{array}{c}
 a \\
 b \\
 c \\
 d \\
 v \\
 w \\
 x \\
 y \\
 z
\end{array}
\end{array} \]

Figure 1 \( H \).

Solution. Each vertex in \( L(H) \) has degree at least 3, while each vertex in \( R(H) \) has degree at most 3. Consequently, the graph is degree-constrained, which by Theorem 11.5.6 implies there is a matching that covers \( L(H) \).

One matching, for example is:

\[ \langle a--z \rangle, \langle b--w \rangle, \langle c--v \rangle, \langle d--x \rangle. \]

Problem 2 (6 points).
An assignment command such as \( w := u + v \) sets the value of variable \( w \) to be the sum of the values of \( u \) and \( v \). Variable values can be stored in the same register if they are not needed at the same time during program execution. The problem of economically allocating registers to store variable values corresponds to a graph coloring problem.
(a) Construct the graph corresponding to the register allocation problem for the following program:

Inputs: $u, v$

\[ w := u + v \]
\[ x := u - v \]
\[ y := w + x \]
\[ z := w - x \]

Outputs: $y, z$

Solution.

(b) Describe a minimal coloring of your graph and its associated assignment of variables to registers.

Solution. Three colors and hence three registers are needed. One possible assignment of variables to registers is indicated in the figure above.