Staff Solutions to Mini-Quiz 6, afternoon

STAFF NOTE: Simple Graphs: Degrees, Isomorphism, Matchings, Mating Ritual, Coloring, Connectivity, Ch.11.1–11.10

Problem 1 (4 points).
Explain why the graph $G$ below has no matching.

Solution. It is not possible because $\{a, b, c, e\}$ is a bottleneck: $|E(G)(\{a, b, c, e\})| = |\{v, x, y\}| = 3 < 4 = |\{a, b, c, e\}|$.

Since the left and right vertex sets of $G$ are the same size, there is a bottleneck on the right iff there is one on the left. So an alternative answer is to observe that $\{w, z\}$ is a bottleneck in the other direction: $|E(G)^{-1}(\{w, z\})| = |\{d\}| = 1 < 2 = |\{w, z\}|$.

Problem 2 (6 points).
An assignment command such as $w := u + v$ sets the value of variable $w$ to be the sum of the values of $u$ and $v$. Variable values can be stored in the same register if they are not needed at the same time during program execution. The problem of economically allocating registers to store variable values corresponds to a graph coloring problem.

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(a) Construct the graph corresponding to the register allocation problem for the following program:

Inputs: $u, v$
- $w := u + v$
- $x := u - v$
- $y := w + x$
- $z := w - x$
- $a := y + z$
- $b := y - z$

Outputs: $y, z, a, b$

Solution.

(b) Describe a minimal coloring of your graph and its associated assignment of variables to registers.

Solution. Four colors and hence four registers are needed. One possible assignment of variables to registers is indicated in the figure above.