Staff Solutions to Mini-Quiz 1

Problem 1 (10 points).
Prove that \( \log_9 12 \) is irrational.

Solution. Proof. Suppose to the contrary that

\[
\log_9 12 = \frac{m}{n}
\]

for some integers \( m, n \) where \( n > 0 \). So we have

\[
\begin{align*}
\log_9 12 &= m/n, \\
9^{\log_9 12} &= 9^{m/n} & \text{(raising 9 to equal powers),} \\
12 &= 9^{m/n} & \text{(def of \( \log_9 \)),} \\
12^n &= 9^m & \text{(raising both sides to the \( n \)th power).}
\end{align*}
\]

But the left hand side of (1) is even since \( n > 0 \), and the right hand side is not.

This contradiction implies that \( \log_9 12 \) must be irrational. ■