

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Directed Acyclic Graphs (DAGs)

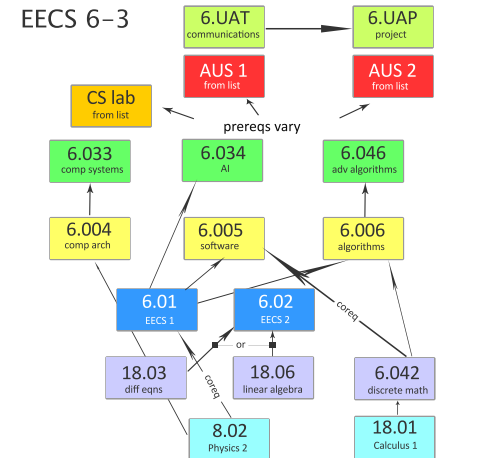


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DA6.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

project: 1/2+1/2
advanced: 2
lab: 1
header: 3
foundational: 3
introductory: 2
math: 2
18.03/06 + 6.042



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DA6.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

indirect prerequisites

u is indirect prereq of v
when there is a sequence
of prereq's from u to v



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scheduling.4

6	9	13	7
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indirect prerequisites

u is an indirect prereq of v
just means that there is a
positive length walk from
 u to v in the 6-3 digraph D :

$$u D^+ v$$



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scheduling.5

6	9	13	7
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Closed walks

Closed walk starts & ends at the same vertex.

How long does it take to graduate if there is a **closed walk starting at 6.042?**

(There is a faculty committee that checks for this kind of thing.)



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DA6.6

6	9	13	7
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Cycles

A **cycle** is a closed walk whose only repeated vertex is its start. A vertex alone is a **length-0 cycle**

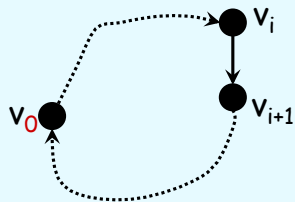
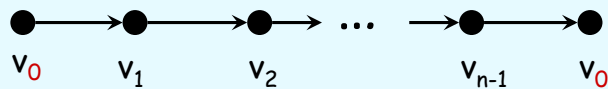


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DA6.7

6	9	13	7
12		10	5
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15	8	11	2

Cycles



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DA6.8

6	9	13	7
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15	8	11	2

Closed Walks

Lemma: Shortest positive length closed walk from v is positive length **cycle** from v !

Proof: like shortest walk is a path



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DA6.9

6	9	13	7
12		10	5
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Directed **A**cylic **G**raph
DAG
 has no positive
 length **cycle**
 (or closed **walk**)



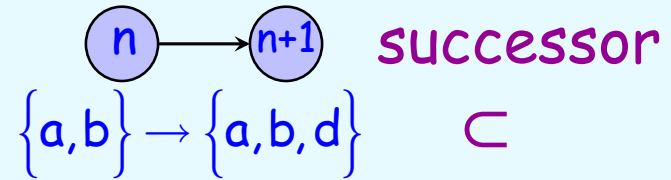
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DA6.10

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Directed **A**cylic **G**raph
 examples:

prerequisite graph

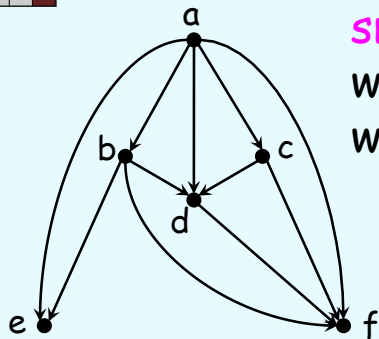


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DA6.11

6	9	13	7
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15	8	11	2

DAG walk relation



smallest DAG
 with same
 walk relation?

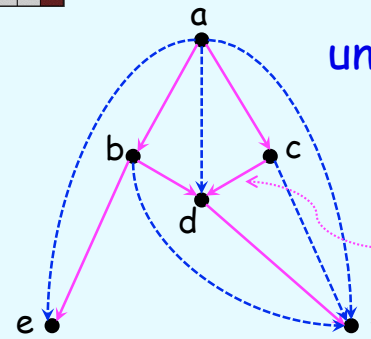


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DA6.12

6	9	13	7
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3	1	4	14
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Covering Edges



unneded edges

any path from
 c to d must
 use $c \rightarrow d$



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DA6.13