**Problem 1.** Here is a simple recursive definition of the set, *E*, of even integers:

**Definition.** Base case:  $0 \in E$ .

**Constructor cases**: If  $n \in E$ , then so are n + 2 and -n.

Provide similar simple recursive definitions of the following sets:

- (a) The set  $S ::= \{2^k 3^m 5^n \mid k, m, n \in \mathbb{N}\}.$
- (b) The set  $T ::= \{2^k 3^{2k+m} 5^{m+n} \mid k, m, n \in \mathbb{N}\}.$
- (c) The set  $L ::= \{(a, b) \in \mathbb{Z}^2 \mid 3 \mid (a b)\}.$

Let L' be the set defined by the recursive definition you gave for L in the previous part. Now if you did it right, then L' = L, but maybe you made a mistake. So let's check that you got the definition right.

(d) Prove by structural induction on your definition of L' that

 $L' \subseteq L.$ 

(e) *Optional: come back to this part only if you finish all the remaining problems.* Confirm that you got the definition right by proving that

 $L \subseteq L'$ .

(f) Optional: come back to this part only if you finish all the remaining problems. Give an unambiguous recursive definition of *L*.