Problem 1. Here is another exciting 6.042 game that's surely about to sweep the nation!

You begin with a stack of n boxes. Then you make a sequence of moves. In each move, you divide one stack of boxes into two nonempty stacks. The game ends when you have n stacks, each containing a single box. You earn points for each move; in particular, if you divide one stack of height $a + b$ into two stacks with heights a and b, then you score ab points for that move. Your overall score is the sum of the points that you earn for each move. What strategy should you use to maximize your total score?

As an example, suppose that we begin with a stack of $n = 10$ boxes. Then the game might proceed as follows:

Stack Heights										**Score**
$\underline{10}$										
5	$\underline{5}$									25 points
$\underline{5}$	3	2								6
$\underline{4}$	3	2	1							4
2	$\underline{3}$	2	1	2						4
$\underline{2}$	2	2	1	2	1					2
1	$\underline{2}$	2	1	2	1	1				1
1	1	$\underline{2}$	1	2	1	1	1			1
1	1	1	1	$\underline{2}$	1	1	1	1		1
1	1	1	1	1	1	1	1	1	1	1
							Total Score		=	45 points

On each line, the underlined stack is divided in the next step.

(a) Can you find a better strategy? Experiment with a few strategies, and before looking at the next page, see if your team can guess what's going on.

(b) As you may have guessed, the strategy is irrelevant: the score is determined solely by the number of boxes. Confirm this using strong induction to prove that the predicate

$$S(n) ::= \text{every way of unstacking } n+1 \text{ blocks gives a score of } (n+1)n/2$$

holds for all $n \in \mathbb{N}$.