Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Handout 32
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Problems for Recitation 7 (3/20/98)

Problem 1. **(a)** How many sets of size one *or* two are contained in the set of integers $0, \ldots, n-1$? *Hint*: There are n^2 ordered pairs of integers $0, \ldots, n-1$, of which all but n have distinct elements.

(b) Suppose $n > 1$ and we place checkers on exactly $n+1$ squares of the $n \times n$ checkerboard. Show that some length occurs more than once as the distance between two different checkered squares. (More precisely, there are checkered squares $S1 \neq S2$ and $S3 \neq S4$ such that the distance between $S1$ and $S2$ is the same as the distance between $S3$ and $S4$, but $\{S1, S2\} \neq \{S3, S4\}$.)

Problem 2. **Claim**: In any sequence of n different numbers, there is either an increasing subsequence of length $> \sqrt{n}$ or a decreasing subsequence of length $\geq \sqrt{n}$.

For example

$$647912538$$

has the decreasing sequence

$$641$$

and the increasing sequence

$$1238.$$

That is, it has "long" subsequences of both kinds.

Consider some sequence of n numbers. To prove the claim, say for numbers a, b in the sequence, that $a \preceq b$ iff either $a = b$, or else both $a < b$ and also a comes before b in the sequence.

(a) Verify that $a \preceq b$ is a partial order on the elements of the sequence.

(b) What kind of subsequences are chains in this poset? ... antichains? Prove it.

(c) Prove the Claim.