Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Problems for Recitation 5 (3/6/98)

Problem 1. We begin with two large glasses. The first glass contains a pint of water, and second contains a pint of wine. We pour 1/3 of a pint from the first glass into the second, stir up the wine/water mixture in the second glass, and then pour 1/3 of a pint of the mix back into the first glass. We now repeat this pouring back-and-forth process a total of n times.

(a) Describe a closed form formula for the amount of wine in the first glass after n back-and-forth pourings.

(b) What is the limit of the amount of wine in each glass as n approaches infinity?

Problem 2. Asymptotic notations

(a) Give an example of functions $f, g : \mathbb{N} \to \mathbb{N}$ such that neither $f = O(g)$ nor $g = O(f)$. Prove it.

(b) Let $f_1, f_2, \ldots$ be a sequence of functions from $\mathbb{N}$ to $\mathbb{N}$ such that $f_i(n) = O(1)$ for all $i \geq 1$. Prove or give a counterexample:

$$\sum_{i=1}^{n} f_i(n) = O(2^n).$$