Problems for Recitation 4

Team Problems

Problem 1. Fast multiplication by adding and doubling.

Consider the following multiplication algorithm, known as the "Russian peasant's multiplication" to multiply numbers $x, y \in naturals$.

The accumulator for the answer is register `a`; registers `r`, `s` are for temporary storage.

```
r:=x; s:=y; a:=0;
Do until s=0:
 if even(s) then {r := 2r; s := s/2};
 if odd(s) then {a := a + r; r := 2r; s := (s-1)/2};
```

The answer xy will be left in the accumulator `a` when the procedure terinates.

Here is an example of the execution of the algorithm.

x	y	r	s	a
5	9	5	9	0
		10	4	5
		20	2	5
		40	1	5
		80	0	45

Model the computation as a state machine. Prove it terminates using a derived variable. Prove that it is correct using an invariant.

Problem 2. Instability in Unisex Marriage.

We might better call this the Buddy Pairing Problem: an even number of men, each with a preference ranking of all the others, to be paired as "buddies" in a stable way.

Derive an example of preferences for this unisex version of the mating problem which shows that it is *not* possible to always pair all the players so that the marriages satisfy the stability criterion. *Hint*: Consider just four men.

Problem 3. The judge says to the guilty defendant,

> "Here is what is called a *Petition Process*:
>
> > You may name a number of times you may petition me. Your petitions may request a number of times you may start a count of days till your next petition or until this whole petition process shall end. Your petitions will be granted. When the number of days named in your last petition is reached, the process will end.
>
> Now you may request a number of petition processes in which you would like to participate. Your request will be granted, but at the end of the last process, your head will be cut off."

The defendant responds, "I want to participate in 2 processes." The judge says "OK", and the defendant asks to submit 4 petitions. The judge says "OK," and the defendant files a petition asking for 10 days. The judge says "OK." 10 days later, the defendant files a petition asking for 20 days. 20 days later the defendant asks for 30 days, and 30 days later for 40 days.

40 days later, he starts his second petition process, this time asking to submit 3 petitions. The first petition is for 100 days. 100 days later, he asks for 200 days. 200 days later he asks for 300 days. 300 days later his head gets cut off `:-(`.

Of course a smarter prisoner could have named much larger numbers and lived out his natural lifetime. But for the sake of philosophy, suppose the prisoner (and the judge and executioner) will live as long as it takes to get executed. Prove that the prisoner will always eventually get executed.

Problem 4. Consider the Stable Marriage Problem when there are more men than women. Define what a set of stable marriages should mean in the presence of bachelors. Show that the Morning-Afternoon-Evening Protocol described in lecture solves this Marriage Problem.