Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professor Albert R. Meyer

Handout 35
March 31, 1998, typo 4/10

Problem Set 7

Due: April 7.

Reading: Rosen §4.1–3, 4.6, 5.4–5

Problem 1.

Calculate the number of ways to place 11 *indistinguishable* balls in four *distinguishable* boxes so that each box contains at least one, but no more than four, balls.

Problem 2.

Prove that

$$\sum_{a=0}^{n} \sum_{b=0}^{n-a} \frac{n!}{a!b!(n-a-b)!} = 3^n .$$

[Hint: find a formula for $(x+y+z)^n$ and then set $x = y = z = 1$.]

Problem 3.

A positive integer is called *square-free* if it is not divisible by the square of any positive integer greater than 1. For example $35 = 5 \cdot 7$ is square-free but $18 = 2 \cdot 3^2$ is not. 1 is square-free. Find the number of square-free positive integers less than 150 (150 included.)

Problem 4.

How many ways can 4 balls be placed in 4 urns if

(a) both the balls and the urns are distinguishable;

(b) the balls are distinguishable but the urns are not;

(c) the urns are distinguishable but the balls are not;

(d) neither the balls nor the urns are distinguishable.

Problem 5.

How many ways can a photographer at a wedding arrange 5 people in a row from a group of 10 people that includes the bride and groom if

(a) The bride must be in the picture?

(b) The bride and the groom must be in the picture?

(c) Exactly one of the bride and groom must be in the picture?

Problem 6.

Six women and nine men are on the faculty of a school's EECS department. The individuals are distinguishable. How many ways are there to select a committee of 5 members if at least 1 woman must be on the committee?

Problem 7.

Let $a_1, \ldots, a_n$ be n positive integers ($n \geq 2$) such that $1 \leq a_i \leq \lfloor (2^n - 2)/n \rfloor$ for all $i = 1, \ldots, n$. Prove that there are two disjoint non-empty subsets $S, T \subseteq \{1, \ldots, n\}$ such that

$$\sum_{i \in S} a_i = \sum_{i \in T} a_i \ .$$

Problem 8.

A shelf holds 13 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen?