Problem Set 5

Due: Tuesday, March 10.

Reading assignment: Rosen 1.7–8; 6.042 Fall97 Notes for Lectures 8 and 9.

Problem 1. A college loan officer offers prospective M.Eng students the following deal. He will give them \$23,100 at the start of each year for five years starting today to pay for their college tuition. He offers the following plan for repayment:
Wait five years, then repay $\$x.00$ at the start of each year for the next ten years.

Suppose the annual interest rate is 8%. What value of x will make the present value of the repayment equal to the present value of the loan?

Problem 2. **(a)** Find a closed form solution for

$$\sum_{k=1}^{n} 1/k(k+1)$$

Hint: Use the fact that $1/k(k+1) = 1/k - 1/(k+1)$.

(b) Use induction to verify your closed form.

Problem 3. Find a closed form solution for

$$\prod_{i=1}^{n} 2 \cdot 4^i$$

Problem 4. Let $0 < x < 1$ and $0 < y < 1$. Evaluate the following infinite sums.

(a)

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} x^i y^{i+2j}.$$

(b)

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} ij x^i y^j.$$

Problem 5. Justify your answers to the following:

(a) Is $2^n = O(4^n)$?

(b) Is $2^{2n} = O(2^n)$?

(c) Is $2^{n+1} = O(2^n)$?

(d) Is $\log_{10} x = \Theta(\log_2 x)$?

Problem 6. Find a Θ bound for $\sum_{k=1}^{n} k^6$.

Problem 7. Prove or give a counterexample: If $f(n) = \Theta(n)$, then $\sum_{i=1}^{n} f(i) = \Theta(n^2)$.

Problem 8. **(a)** Find a closed form solution for $\sum_{i=1}^{n} i^2 x^i$.

(b) Verify your closed form by induction.

Problem 9. Let $S_n = \sum_{i=1}^{n} i^{1/3}$. Use the integral method to determine

$$\lim_{n\to\infty} \frac{S_n}{n^{4/3}}$$