Massachusetts Institute of Technology | Handout 11
6.042J/18.062J: Mathematics for Computer Science | February 18, 1998
Professor Albert R. Meyer

Problem Set 3

Due: Tuesday, February 24.

Reading assignment: Rosen sections 7.1–7.4, 7.8, 8.1–2

Problem 1. [10 pts]

The graphs C_n, Q_n, W_n, and $K_{n,m}$ are defined in Rosen 7.2.

(a) Describe the adjacency matrix for C_n.

(b) Describe the adjacency matrix for $K_{n,m}$.

(c) Give a simple recursive definition of the adjacency matrix for the graph Q_{n+1}. *Hint:* Use the matrix of Q_n and suitable identity matrices.

Problem 2. [10 pts]

Find the minimum number of colors to color (with no two adjacent vertices given the same color, *cf.* Rosen, 7.8) each of C_9, C_{20}, W_{13} and $K_{311,222}$. Explain briefly.

Problem 3. [20 pts]

(a) Prove or disprove that pair of graphs given in Rosen 7.3, Exer. 38 are isomorphic.

(b) Prove or disprove that pair of graphs given in Rosen 7.3, Exer. 42 are isomorphic.

Problem 4. [30 pts]

Some properties of a simple graph, G, are described below. For each property, carefully prove or give a counterexample to the claim that the property is *invariant under isomorphism* (*cf.* Rosen, p.455).

(a) G has an even number of vertices.

(b) None of the vertices of G is an even integer.

(c) G has a vertex of degree 3.

(d) G has exactly one vertex of degree 3.

(e) One of the vertices of G is also an edge of G.

Problem 5. [30 pts]

Definition: A simple graph, G, has *inherited degree* $d \geq 0$ if and only if
(i) every vertex of G has degree $\leq d$, or
(ii) there is a vertex $v \in G$ of degree $\leq d$, and the subgraph of G obtained by removing v has inherited degree $\leq d$.

(a) Prove that W_n has inherited degree 3.

(b) Prove that a tree has inherited degree 1.

(c) Prove that if a graph has inherited degree d, then it is $(d+1)$-colorable.

(d) For every $n > 0$, describe an example of a graph which does not have inherited degree n, but which is 2-colorable.