Problem Set 1

Due: In lecture, Tuesday, February 10.

Reading assignment: Sections 1.1–1.5, 2.3, and 3.1 of Rosen. Notes for Lecture 1 and Recitation 1. Start *The Nuts and Bolts of Proofs* aiming to finish before the third week of class.

Problem 1.

(a) The *symmetric difference*, $A \oplus B$, of two sets, A and B, is defined by

$$A \oplus B = (A \cup B) - (A \cap B).$$

Show that $A \oplus B = (A - B) \cup (B - A)$.

(b) Prove that $\oplus$ is associative.

Problem 2. Show that each of the following implications is a tautology by using truth tables.

(a) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

(b) $[p \wedge q \wedge (p \rightarrow r)] \rightarrow (q \wedge r)$

Problem 3. Find a compound proposition involving the propositions p, q, and r that is true exactly when *both*
(i) at least one of the propositions is true, *and also*
(ii) at least one of the propositions is false.
The compound proposition is false otherwise.

Problem 4. Let $W(x, y)$ be the statement "x was one of the writers of y," and $S(x)$ be the statement "x is a professional screenwriter," where the universe of the discourse for x is the set of all people and for y is the set of all movies. Express each of the following sentences in terms of $W(x, y), S(x)$, quantifiers, and logical connectives,

a) Someone who helped write "Titanic" was not a professional screenwriter.

b) Every movie had a writer who was a professional screenwriter.

c) There is a professional screenwriter who has written "Pulp Fiction" and "Jackie Brown."

d) There is only one movie which had exactly one writer who was not a professional screenwriter. (Don't introduce numbers. You will have to add more standard variables ranging over people and over movies).

Problem 5. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x (P(x) \vee Q(x))$ are not logically equivalent.

Problem 6. Define predicates and state the following arguments as predicate logic formulas. Prove the correct arguments and show the incorrect ones are false.

(a) Fairies are never fierce; some dragons are not fierce. Thus, some dragons are not fairies.

(b) Gold is a liquid asset, and he will be satisfied only with gold. Thus, no fixed asset will satisfy him.

Problem 7. Cast the following into propositional symbols, and explain whether Gandalf can be a Hobbit.

> Careless and weak people are not Wizards. People who are not careless are cautious. Cautious people don't eat mushrooms. Weak people are not powerful. Hobbits are not powerful and eat mushrooms. Gandalf is a Wizard.

Problem 8.

(a) Prove by contradiction: for all integers $n > 1$, the nth root of a prime number is irrational.

(b) Your proof must depend on some property of prime numbers not shared by all integers greater than one. Why? What property?

Problem 9. Prove by cases: for every $n \in \mathbb{N}$, either $4 | n^2$ or $4 | (n^2 - 1)$.