Massachusetts Institute of Technology — Tutorial 1
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Problems for Tutorial 1

1 The Superman Problem

Cast the following into propositional symbols, and verify or refute:

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent. If he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

2 A Touch of Group Theory

A *group* is a set G together with a binary operation $\cdot : GxG \to G$ that satisfies the following axioms.

(a) $\forall a, b, c \in G, (a \cdot b) \cdot c = a \cdot (b \cdot c)$.

(b) $\exists e \in G$ (called the *identity*) such that $\forall a \in G, a \cdot e = e \cdot a = a$.

(c) $\forall a \in G, \exists a^{-1} \in G$ (called the *inverse* of a), such that $a \cdot a^{-1} = a^{-1} \cdot a = e$.

Using the three group axioms prove the following propositions.

1. Show, by contradiction, that the identity element is unique.
2. Show that for every $a \in G$, the inverse a^{-1} is unique.

   Hint: First show the following lemma:

   (a) $\forall a, b, c \in G, a \cdot b = a \cdot c \Rightarrow b = c$

   (b) $\forall a, b, c \in G, b \cdot a = c \cdot a \Rightarrow b = c$
3. $\forall a \in G, (a^{-1})^{-1} = a$
4. $\forall a, b \in G, (a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$