Problems for Tutorial 10

Problem 1. N people leave their hats in a coatroom. On the way out the person in charge of the coatroom hands their hats back to them in an arbitrary order. Use inclusion-exclusion to count the total number of ways in which the hats can be handed back to them such that no person gets his/her own hat back.

Problem 2. Consider playing the *Lets Make a Deal*, where there are three prizes: a “Gift Certificate” from Toscanini, a “Nerd Pocket Protector”, and a “Nerd Pride Button”. Your goal is to win the gift certificate.

(a) Suppose you (the contestant) happen to know that Carol always reveals the “Nerd Pocket Protector”, if you did not choose the box covering it.
Does this additional information change the odds of winning for you? Is there a better strategy? By how much (if any) does this information increase your chances of winning?

(b) You are playing according to the original assumptions, but before you make your initial choice, you happen to see Carol placing a “Nerd Pride Button” underneath Box 1. What should you do to try to win the prize (the ice cream)? What is the probability that you win?

Problem 3. Consider playing the *Lets Make a Deal* game with the following additional information.

(a) Suppose Monty is determined to make you lose. In particular, if you initially choose a losing door, he decides to end the game right there, opening your door to reveal the nerd item you have “won”, never giving you the chance to switch doors. What is the probability of winning if you play the “don't-switch strategy”? What if you play the “switch strategy”?

(b) Now suppose that if you initially choose a loosing door, Monty decides to end the game with (known) probability p. What is your optimal strategy for each value of p? What is the corresponding probability of winning?

(c) Suppose the scenario is the same as in the previous part, but you are using randomized strategy. In particular, you decide to switch with probability p_s. Can you do better? What should the value of p_s be for every value of p?