Problem Set 8

Due: November 4.

Reading: 4.1, 4.3, 4.6, 5.4, 5.5, 9.1-9.3 of Rosen

Problem 1. [**10 points**]

Calculate the number of ways to place 10 indistinguishable balls in four boxes with at least one in each box and at most four in each box.

Problem 2. [**10 points**]

(a) Show that

$$\sum_{i=0}^{r} \binom{n+i}{i} = \binom{n+r+1}{r}$$

(b) Find a formula for the number of ways a $2n+1$ game world series can be played, that terminates after one team wins $n+1$ games. The two teams are distinguishable. Use part (a) to write your answer in closed-form.

Problem 3. [**10 points**]

How many ways can 4 balls be placed in 3 urns if

(a) both the balls and the urns are distinguishable;

(b) the balls are distinguishable but the urns are not;

(c) the urns are distinguishable but the balls are not;

(d) neither the balls nor the urns are distinguishable.

Problem 4. [**10 points**]

(a) Design a huffman code for the following 8-letter alphabet with frequency counts as follows:

A .45
B .01

C .16
D .03
E .05
F .07
G .06
H .17

(b) Give the code for the string "AHAACDEB".

(c) How many bits are needed to encode a message with 100 letters, containing 45 A's, 1 B, 16 C's, 3 D's, 5 E's, 7 F's, 6 G's and 17 H's using this code?

Problem 5. [10 points]

How many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people that includes the bride and groom if

(a) The bride must be in the picture?

(b) The bride and the groom must be in the picture?

(c) Exactly one of the bride and groom must be in the picture?

Problem 6. [10 points]

Seven women and nine men are on the faculty of a school's math department. The individuals are distinguishable. How many ways are there to select a committee of 5 members if

(a) at least 1 woman must be on the committee?

(b) at least 1 man and 1 woman must be on the committee?

(c) there must be more men than women on the committee?

Problem 7. [10 points]

(a) Describe a method for guessing 2 cards when 7 out of 9 are revealed.

(b) Prove that you cannot guess 2 cards when 4 out of 6 are revealed.

Problem 8. [10 points]

A shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen?

Problem 9. **[10 points]**

Compute the (ordinary) generating function for each of the following sequences.

(a) $\langle 1, 1, 1, 1, 1, \ldots \rangle$

(b) $\langle 1, 1, 1, 0, 0, 0, 0, 0, 0, \ldots \rangle$

(c) $\langle 0, 0, 1, 1, 1, 1, 1, \ldots \rangle$

(d) $\langle 0, 0, 0, 1, 1, 1, 1, 1, \ldots \rangle$

(e) $\langle 0, 0, 0, 1, 1, 2, 1, 1, 1, 1, 1 \ldots \rangle$

(f) $\langle 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, \ldots \rangle$

(g) $\langle 1, 4, 9, 16, 25, \ldots \rangle$

(h) $\langle a_k \rangle = \langle k(k+2) \rangle$

(i) $\langle a_k \rangle = \left\langle 3^k + 5^k + 7^k \right\rangle$

(j) $\langle a_k \rangle = \langle 1/k! \rangle$

Problem 10. **[10 points]**

Let S be the multiset $\{\infty \cdot a_1, \infty \cdot a_2, \infty \cdot a_3, \infty \cdot a_4\}$. For each of the following problem parts, determine the generating function for the sequence $\langle g_0, g_1, \ldots \rangle$, where g_r is the number of r-combinations of S with the specified added restriction.

(a) Each a_k occurs an odd number of times.

(b) Each a_k occurs a multiple of 3 times.

(c) The element a_1 does not occur, and a_2 occurs at most once.

(d) The element a_1 occurs 1, 3, or 11 times, and the element a_2 occurs 2, 4, or 5 times.