Problem Set 1

Due: Start of Class, Tuesday, September 9.

Reading Assignment:
Read all of "The Nuts and Bolts of Proofs" during the first 2 weeks of class.
Read 1.1-1.5 and 2.3 of Rosen as background for the course.
Read 3.1 of Rosen.

Problem 1. **(a)** Establish the following set identity
$(B - A) \cup (C - A) = (B \cup C) - A$

(b) The *symmetric difference* of two sets is defined by $A \oplus B = (A - B) \cup (B - A)$. Show that $A \oplus B = (A \cup B) - (A \cap B)$.

Problem 2. Show that each of the following implications is a tautology by using truth tables.

(a) $[\neg p \wedge (p \vee q)] \rightarrow q$

(b) $[p \wedge (p \rightarrow q)] \rightarrow q$

(c) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Problem 3. Find a compound proposition involving the propositions p, q, and r that is false when exactly two of p, q, and r are false. The compound proposition is true otherwise.

Problem 4. Let $P(x, y)$ be the statement "x wrote y." Express each of the following sentences in terms of $P(x, y)$, quantifiers and logical connectives, where the universe of the discourse for x is the set of all poets and for y is the set of all poems.

a) There is a poet who wrote "The Raven"

b) Every poet has written a poem.

c) Not every poem has been written by a poet.

d) There is a poet who has written "The Lady of Shallot" and the "Lotus Eaters"

e) At least two poets have written a poem with the name "Ulysses" (Don't introduce numbers. You will have to add one more standard symbol for this one).

Problem 5. Show that $\exists z \forall x \exists y Q(x, y, z)$ implies but is not equivalent to $\forall x \exists y \exists z Q(x, y, z)$.

Problem 6. Define predicates and state the following arguments as predicate logic formulas. Prove the correct arguments and show the incorrect ones are false.

(a) No lemons are fierce; some dragons are not fierce. Thus, some dragons are not lemons.

(b) Gold is heavy; nothing but gold will silence him. Thus, nothing light will silence him.

Problem 7. Cast the following into propositional symbols and answer the question.

Careless and weak people are not Wizards. People who are not careless are cautious. Cautious people don't eat mushrooms. Weak people are not powerful. Hobbits are not powerful and eat mushrooms. Gandalf is a Wizard. Can Gandalf be a Hobbit?

Problem 8. Prove that a positive integer n is a perfect square if and only if there are an odd number of positive integers (not necessarily just primes) that divide n. For example, there is one positive integer that divides 1.

Problem 9. Prove by contradiction: the square root of an irrational number is irrational.

Problem 10. You have a set of 5 identical-looking balls, and you are told that there are only two possible values for the weight of each ball. You are allowed to compare the weights of (any) two balls (but only two at a time). There are two possible outcomes of a comparison: "equal" or "not equal" (*not* which ball is heavier). Can you find a ball with the same weight as the majority of the balls in three comparisons? (For example if all five balls have the same weigh, then any ball will do).