Problem Set 10

Due: November 21, 1995.

Reading: Pages 261-275 of *Discrete Mathematics and its Applications* by Kenneth H. Rosen.

Problem 10-1. Consider playing the *Lets Make a Deal* game with the following additional information.

(a) You are playing according to the original assumptions, but before you make your initial choice, you happen to hear Andrew (Carol) placing a "Nerd Pride" pocket protector underneath Box 1. What should you do to try to win the prize (the ice cream)? What is the probability that you win?

(b) Consider the game where Monty always puts the prize behind the same door (say box 1) but you pick your doors uniformly at random. Show that this version of the game is the same as the original one where both Monty and you picked the doors at random. (By "same as" we mean that the strategies "don't-switch," "switch" have the same probabilities of winning.)

(c) Suppose Monty is determined to make you lose. In particular, if you initially choose a losing door, he will decide to end the game right there, opening your door to reveal the goat you have "won", never giving you the chance to switch doors. What is the probability of winning if you play the "don't-switch strategy"? What if you play the "switch strategy"?

Problem 10-2. **(a)** Suppose you are playing the *Lets Make a Deal* game with 4 doors rather than 3. Say Monty and you choose doors according to the uniform distribution. After your initial choice, Monty reveals a loser and offers you the chance to switch. Subsequently, he reveals another loser and again offers you the chance to switch. Compute the probability of winning for each of the four possible strategies: (1) stick both times, (2) switch both times, (3) stick the first time, switch the second time, and (4) switch the first time, stick the second time. Which strategy is the best?

(b) Consider the 3 door version of the game. Suppose you know that Monty always reveals the lowest numbered door available. Does this new information help you? If not, why not? If yes, what should your strategy be and why?

Problem 10-3. A witness to a crime says that the criminal had a beard. We know from experience that if a criminal has a beard, witnesses will say so with probability 0.8, and if the criminal does not have a beard, witnesses will say that he does with probability 0.12. If 15% of the population has beards, what is the probability that the criminal in this case has a beard (given that the criminal was drawn uniformly at random from the entire population)?

Problem 10-4. Suppose A, B, C are independent events. Show that any of the combinations

$$A^c, B, C;\ A, B^c, C; \cdots; A^c, B^c, C; \cdots; A^c, B^c, C^c$$

are also independent. Furthermore, show that A and $B \cup C$ are independent.

Problem 10-5. Once upon a time, a soldier was sentenced to death. To show his kindness, the king decided to give him another chance. He gave the soldier 50 black balls and 50 white balls and 2 pots and told him to divide the balls between the two pots in whatever way he liked. Then the king would come back and randomly take away one pot and mix the balls in the other. The soldier would then pick a ball from the remaining pot and live if and only if it was white. What strategy should the soldier take? Prove that it is the best.

Problem 10-6. Mr. James W. Pitman received a letter:
" $\cdots$ to inform you that one of the following three people has won a New Datsun Sentra: Collin Andrus; James W. Pitman; Larry Abbott. $\cdots$"
" $\cdots$ you will receive one of the following for sure:
(1) hairdryer; (2) opportunity to be a partner in Bonanza Gold Mine Inc; (3) Datsun Nissan Sentra.
To claim your gift, call $\cdots$"
The fine print at the back of the letter says "your chances of winning a hairdryer or a Datsun Nissan Sentra are 1 in 10,000 and your chances of winning an opportunity to be a partner in Bonanza Gold Mine Inc. are 9,998 in 10,000."

Let us assume that this is an honestly conducted sweepstakes, with 10,000 participants and exactly one Datsun being awarded. Now that Mr. Pitman knows that the winner is one of three individuals (and he is on the list), the rules of conditional probability imply that he has a $\frac{1}{3}$ chance of winning the Datsun. True or False? Explain!