Massachusetts Institute of Technology | Handout 34
6.042J/18.062J: Mathematics for Computer Science | October 24, 1995
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Problem Set 7

Due: October 31, 1995.

Reading: Sections 4.1–4.3,4.6 and 4.7 of *Discrete Mathematics and its Applications* by Kenneth H. Rosen.

Problem 7-1. A poker hand consists of 5 cards drawn from a standard deck of 52 cards. How many different poker hands are there that contain

(a) any set of 5 cards?

(b) a royal flush (10, Jack, Queen, King and Ace of one suit)?

(c) 4 of a kind (e.g. four 7's)?

(d) a full house (two of a kind and three of another kind)?

(e) a flush (5 cards in a suit)?

(f) a straight (five cards in a sequence without any suit restrictions)?

(g) 3 of a kind?

(h) 2 pairs (e.g. two 7's and two 9's)?

(i) 1 pair?

Problem 7-2. On a set of n elements how many of the following are there

(a) relations?

(b) symmetric relations?

(c) reflexive relations?

(d) symmetric and reflexive relations?

(e) symmetric or reflexive relations?

(f) equivalence relations (this problem is for extra credit)?

Problem 7-3. Morse code is a system for representing an alphabet with dots and dashes. The arrangements of dots and dashes is then transmitted using eletrical pulses of varying lengths or analogous mechanical or visual signals. For example, the letter A may be represented by dash-dot-dot and the letter B by dash-dot. Since it takes time and effort to transmit, the code must be as compact as possible. For a compact code what is the maximum number of dots/dashes required by any character if we wish to represent

(a) the letters A,B,. . .,Z?

(b) the letters A,B,. . .,Z,a,b,. . .,z,0,1,. . .,9?

(c) Given that two dots in a row can get confused with a dash, how many different characters can you represent with k dots/dashes that have no consecutive dots?

(d) Suppose a dot takes 1 seconds, a dash takes 3 seconds, and each must be followed by a 1 second interval of silence. How many morse symbol sequences have sending times less than k seconds (including the last interval of silence)?

Hint: For parts c) and d) determine and solve a recurrence conditioning on the value of the first symbol.

Problem 7-4. Prove that

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$$

whenever n and r are positive integers

(a) using a combinatorial argument.

(b) using Pascal's identity.

Problem 7-5. Find the coefficient of

(a) x^{10} in $(x + (1/x))^{100}$.

(b) x^8y^9 in $(3x + 2y)^{17}$.

(c) x^k in $(x^2 - (1/x))^{100}$.

Problem 7-6. Prove that

$$\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}$$

(a) using a combinatorial argument.

(b) by finding a closed form for $\sum_{k=1}^{n} \binom{n}{k} kx^k$ and evaluating it at $x = 1$.