Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professor David Karger

Tutorial 9
April 14, 1997

Problems for Tutorial 9

Problem 1. Find a $\Theta(\cdot)$ bound for

$$T(n) = T(\sqrt{n}) + \lg \lg n.$$

The base condition is $T(1) = 1$.

Problem 2. Consider the linear recurrence
$T(n) = \sum_{i=1}^{d} a_i T(n/2^i) + G(n)$

- Find a transformation $n \to m$ that will convert this problem io a difference equation of the form
  $f(m) = \sum_{i=1}^{d} a_i f(m-i) + g(m)$

- Apply your results to find the solution of the recurrence relation
  $T(n) = T(n/2) + 2T(n/4) + nlogn$

Problem 3. Compute the following limits.

- $\sum_{i=1}^{\infty} \frac{1}{i(i+1)}$.
- $\lim_{n\to\infty} 1/n + 1/(n+1) + \ldots + 1/2n$
- $\lim_{n\to\infty} \frac{n}{2^n} \sum_{k=1}^{n} \frac{2^k}{k}$