Problems for Tutorial 6

1 Warmups

(a) Determine whether each of the following functions, $f : \mathbb{R} \to \mathbb{R}$, is an injection, surjection or a bijection. Also, give the inverse if it exists.

- $f(x) = e^x$
- $f(x) = x^2 + 3$
- $f(x) = (x^2 + 1)/(x^2 + 2)$

(b) What is the only function on a set A that is also an equivalence relation?

2 Seven Pigeons

Let S be the set of numbers $7, 77, 777, \ldots$. Prove that there is an element of S that is divisible by 1997.

3 Uncomputable Functions

A computable function from the naturals to the naturals is a finite piece of code (in your favourite programming language). The code takes as input a natural number and outputs a natural number. Prove that not all functions f from the naturals to the naturals are computable.

4 The Power of Power Sets

Let G be the set of all functions $f : \mathbb{N} \to \{0, 1\}$.

(c) Prove that $|G| = |2^{\mathbb{N}}|$.

(d) Let U be an uncountable set and C any countable subset of U. Prove that $U - C$ is uncountable.

(e) Prove that $|2^{\mathbb{N}}| = |(0,1)|$ and conclude that $|2^{\mathbb{N}}| = |\mathbb{R}|$.