Massachusetts Institute of Technology | Tutorial 5
6.042J/18.062J: Mathematics for Computer Science | March 10, 1997
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Problems for Tutorial 5

Problem 1. Let R be a relation on a possibly infinite set A. Recall that the powers of a relation are recursively defined by

$$R^1 := R$$

$$R^{n+1} := R^n \circ R.$$

We define the *Transitive Closure* of R to be the relation given by $\bar{R} = \bigcup_{i=1}^{\infty} R^n$
Prove that $\bar{R}$ is transitive.

Problem 2. The Dining Philosophers There are 4 philosophers who sit down to eat dinner at a round table. In front of each of them is a plate of spaghetti and to the right of each of them is a fork. In order to eat, each philosopher must do the following:

1. pick up the fork directly to his left
2. pick up the fork directly to his right
3. eat
4. put down the fork directly to his left
5. pick down the fork directly to his right

(a) Show the state machine for this scenario. Show that it is possible for a series of state transitions to occur such that no philosopher can eat (this is called deadlock).

(b) Detecting a problem in their eating strategy, the philosophers start over. Each philosopher is assigned a unique number 1 through 4. A fork must be used by philosophers in numerical order. For example, if a fork is sitting between philosophers 2 and 4, 4 must wait until 2 has picked up the fork, eaten, and put it down before 4 can use the fork. The philosophers may be seated in any order.

Show the state machine for this scenario. Prove that with any seating arrangement and any order of philosophers, all philosophers will get to finish their dinner.