Problems for Tutorial 4

Problem 1. What does the following procedure compute? Prove it.

something(n, m)
  $r \leftarrow m - n$
  if $(r > 0)$ return$(n +$ something$(n+1, m))$
  else return(0)

Problem 2. Bubble Sort is one of the simplest sorting algorithms. Its goal is to put a list of n numbers in increasing order. Bubble Sort consists of $n-1$ passes through the list. On each pass, whenever an element is encountered that is smaller than the element immediately preceding it in the list, the two elements are interchanged. If we imagine the list as running down a column, the smaller elements will "bubble" upwards as they are interchanged with larger elements. A pseudocode description of Bubble Sort is given below.

BubbleSort$(a_1, ..., a_n)$
  for $i = 1$ to $n-1$
    for $j = 1$ to $n-i$
      if $(a_j > a_{j+1})$ interchange a_j and a_{j+1}

Use induction to prove that Bubble Sort will always terminate with the list sorted in increasing order.

Problem 3. We define a well-balanced string of parentheses to be:

- The empty string, ϵ.
- The string (V)W, where V and W are well-balanced strings of parentheses.

The following procedure is designed to take an input that is a string consisting only of left and right parentheses. If its input is a well-balanced string of parentheses, it is meant to return ϵ. If its input is a string of the form P)W, where P is a well-balanced string of parentheses and W is an arbitrary (possibly empty) string, then it is meant to return the string)W. For any other input, it is meant to return the special value *error*. It uses the auxiliary procedures First-char and Remove-first-char, which return the first character of a string and everything but the first character of a string, respectively. (The strings are presumed to be non-empty in both cases).

Check(w)
  if ($w = \epsilon$) return(ϵ)
  if (First-char(w) = "(")
    $w \leftarrow$ Remove-first-char(w)
    $w \leftarrow$ Check(w)
    if (($w = \epsilon$) or ($w = error$))
      return($error$)
    else
      $w \leftarrow$ Remove-first-char(w)
      $w \leftarrow$ Check(w)
  return(w)

Is the procedure correct? Either prove correctness or provide a counterexample. You should only consider inputs which consist entirely of left and right parentheses.