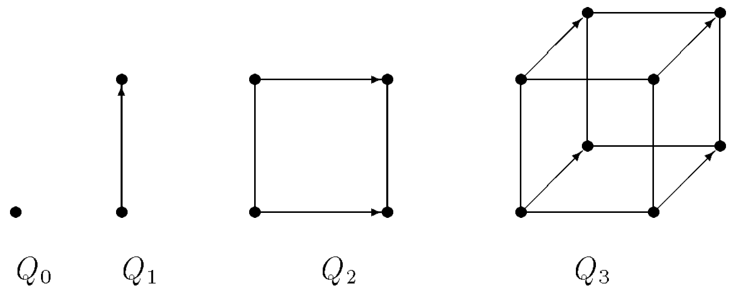
Problems for Tutorial 3

1 Graphs.....

An n-cube, Q_n, is a graph with vertices and edges that can be created in the following way:

Q_0 is one vertex.

Q_n, for $n \geq 1$ is formed by taking an $(n-1)$-cube, Q_{n-1}, making a copy of it, and connecting each pair of corresponding nodes.



A set of vertices in a graph is called *independent* if no two vertices in the set are adjacent. The *independence number* of a graph is the maximum number of vertices in an independent set of vertices for the graph.

a) Show that the number of vertices in Q_n is 2^n.

b) Show that 2^{n-1} is an upper bound for the independence number of Q_n.

c) Prove that Q_n has an independent set of size 2^{n-1} .

2 Uniqueness of Factorization

Prove the following theorem:
Every positive integer can be written as a unique product of primes (where order is irrelevant).

3 A Number Theory Application

Suppose I have a collection of switches numbered 0 through n, all initially off. Someone comes and flips every switch (so they are all on). Another person then flips every second switch, starting with $0, 2, \ldots$. The next person flips every third switch, and so on until the last person flips only the switches numbered 0 and n. Which switches are on at the end? Prove it.