Problem Set 9

Reading: Maurer & Ralston: Sections 6.4–6.7. Tom's lecture notes.

The oral problems are due in tutorial on Monday, May 2. The written problems are due in class on May 3.

Problem O9-1. Derive good upper and lower bounds on the probability that when 1000 fair (and mutually independent) coins are tossed, 400 or fewer will come up heads. Don't forget to include error terms (as was done in recitation) or to sum over the cases when fewer than 400 heads come up (as was done in class). You may use any formulas that were derived in class or in recitation.

Problem O9-2. Let $R : S \mapsto \mathcal{N}$ be a random variable on sample space S. Let $G(z)$ be the probability generating function for R. Show that $Ex(R) = G'(1)$ where G' is the derivative of G with respect to z.

Problem O9-3. Let R and $G(z)$ be as in the previous problem. Show that $G(z)/(1-z)$ is the OGF for the cummulative distribution function for R, i.e., show that $G(z)/(1-z) = \Sigma_{n=0}^{\infty} Pr(R \leq n) z^n$.

Problem O9-4. A couple keeps having babies until they get a child of each sex (whereupon they stop). Each child is equally likely to be a boy or a girl. What is the expected number of children that they will have?

Problem O9-5. Problem 13 on page 490 from Maurer Ralston.

Problem O9-6. Problem 17 on page 491 from Maurer Ralston.

Problem O9-7. Problem 19 on page 490 from Maurer Ralston.

Problem O9-8. Problem 33 on page 492 from Maurer Ralston.

Problem W9-9. Let $f(k) = \binom{n}{k}2^{-n}$. Use Stirling's approximation to show that $f(n/2 - b\sqrt{n}) \sim (\sqrt{2/\pi n})e^{-2b^2}$.

Problem W9-10. Consider the following sequential n-player tournament. In round 1, players 1 and 2 play each other. In round i $(2 \leq i \leq n-1)$, the winner of round $i-1$ plays player i+1 (who is called the *challenger*). Assume that there is a total ranking of the players and that the better player always wins. If all $n!$ permutations of players are equally likely, what is the expected number of times that a challenger will win a round. (There is no challenger in round 1).
Hint: notice that the challenger wins in round i $(i > 1)$ if and only if he/she is the best among the first $i+1$ players. Use indicator variables and linearity of expectation.

Problem W9-11. Consider the following game: in each round you flip a coin, if it comes up heads you gain a dollar, else you lose one. You have one dollar to begin with and the game stops when you go broke. If the coin has probability p of coming up heads show that your probability of going broke is $f(p) = \Sigma_{n=0}^{\infty} 1/(n+1)\binom{2n}{n}p^n(1-p)^{n+1}$. Draw a graph of $f(p)$ versus p and show that when $p = 2/3$ your chance of going broke is even.