Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Handout 6
February 4, 1994

Addendum to Problem Set 1

A few changes have been made in Problem Set number 1 in order to provide you with additional information. Problem O1-9 is reprinted here for your convenience with an addition made in boldface. Problem W1-11 has been entirely rewritten.

Problem O1-9. For any integer $n \geq 3$, consider a simple, not necessarily convex, polygon of n vertices drawn in the plane. (A polygon is **simple** if it does not cross itself.) An ***n*-gon** is a polygon with n vertices. A **triangulation** of an n-gon P is a partition of the interior of P into triangles such that the vertices of any such triangle are vertices of P. **You may assume that any simple polygon has a chord, i.e. a line connecting two vertices of the polygon, which lies in the interior of the polygon and which separates the polygon into two other simple polygons.**

(a) Use induction to show that any triangulation of an n-gon is comprised of exactly $n-2$ triangles.

(b) Using the fact the sum of the interior angles of a triangle is π, give a formula for the sum of the interior angles of an n-gon. Prove that your formula is correct.

Problem W1-11. Consider a simple polygon drawn with vertices on the integer $\mathbf{Z} \times \mathbf{Z}$ lattices, as is shown in Figure 1. Let i be the total number of lattice points in the interior of the polygon, and let b be the total number of lattice points on the boundary.

(a) We say a rectangle is **rectilinearly oriented** if its edges are parallel to the x and y axes. For the polygons considered here, this implies that the edges lie along rows or columns of lattice points. Prove that the area of any rectilinearly oriented rectangle is $i + b/2 - 1$ where again i is the number of lattice points interior to the rectangle, and b is the number of lattice points on the rectangle's boundary.

(b) We say a right triangle is rectilinearly oriented if its two legs are parallel to the x and y axes. Prove that the area of any rectilinearly oriented right triangle is $i + b/2 - 1$. *Hint:* Assume there are r points on the hypotenuse. It turns out that the area calculation may be shown to be independent of r.

(c) Prove that any triangle (with vertices on the lattice) has area $i + b/2 - 1$ by inscribing it in a rectilinearly oriented rectangle.

(d) Prove that any simple lattice polygon has area $i + b/2 - 1$.

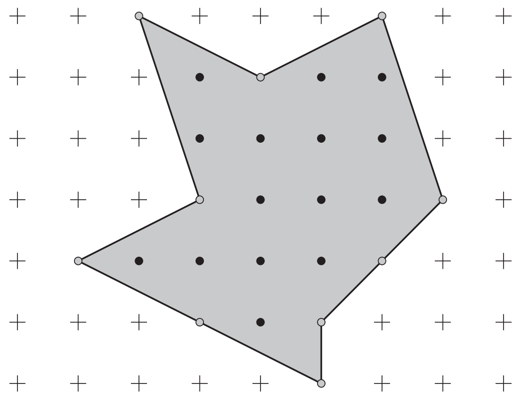

Figure 1: A simple polygon in the lattice $\mathbf{Z} \times \mathbf{Z}$, with 15 lattice points in its interior, 10 lattice points on its boundary and area 19.